ALTERNATE FORM OF THE CUBIC FORMULA – EXAMPLES

Case 1; $c = 0$ and $d = 0$:

Solve $(x-4)^3 = x^3 - 12x^2 + 48x - 64 = 0$ for x.
Solution:

$p = -12$, $q = 48$, $r = -64$.
$c = 0$, $d = 0$.

$x = -\frac{1}{3}p = 4$ (three times) .

Case 2; $c = 0$ and $d \neq 0$:

Solve $x^3 - 5x^2 + \frac{25}{3}x - \frac{341}{27} = 0$ for x.
Solution:

$p = -5$, $q = \frac{25}{3}$, $r = -\frac{341}{27}$.

$c = 0$, $d = -4$.

$A^3 = -2d = 8 = R^3\{\cos 3\Theta + i \sin 3\Theta\}$, $R = 2$, $\Theta = 0°$.

1) $A = R\{\cos(\Theta + 0°) + i \sin(\Theta + 0°)\} = 2$
2) $A = R\{\cos(\Theta + 120°) + i \sin(\Theta + 120°)\} = -1 + \sqrt{3}\, i$
3) $A = R\{\cos(\Theta + 240°) + i \sin(\Theta + 240°)\} = -1 - \sqrt{3}\, i$

1) $x = -\frac{1}{3}p + A = \frac{11}{3}$

2) $x = -\frac{1}{3}p + A = \frac{2}{3} + \sqrt{3}\, i$

3) $x = -\frac{1}{3}p + A = \frac{2}{3} - \sqrt{3}\, i$

Case 3a; $c \neq 0$ and $d^2 + c^3 > 0$:

Solve $x^3 - x^2 + 28x + 290 = 0$ for x.
Solution:

$p = -1$, $q = 28$, $r = 290$.

$c = 9.\overline{2}$, $d = 149.\overline{629}$.

$A^3 = -d - \sqrt{d^2 + c^3} = -301.8576506 = R^3\{\cos 3\Theta + i \sin 3\Theta\}$,

$R = 6.708118551$, $\Theta = 60°$.

ALTERNATE FORM OF THE CUBIC FORMULA – EXAMPLES

1) $A = R\{\cos(\Theta + 0°) \quad + i\sin(\Theta + 0°) \quad \} = \quad 3.354059275 + 5.809401077\,i$
2) $A = R\{\cos(\Theta + 120°) + i\sin(\Theta + 120°)\} = -6.708118551$
3) $A = R\{\cos(\Theta + 240°) + i\sin(\Theta + 240°)\} = \quad 3.354059275 - 5.809401077\,i$

1) $x = -\frac{1}{3}p + A - \frac{c}{A} = \quad 3 + 7\,i$

2) $x = -\frac{1}{3}p + A - \frac{c}{A} = -5$

3) $x = -\frac{1}{3}p + A - \frac{c}{A} = \quad 3 - 7\,i$

Case 3b; $c \neq 0$ and $d^2 + c^3 = 0$:

Solve $x^3 - 11x^2 + 40x - 48 = 0$ for x.
Solution:

$p = -11$, $\qquad q = 40$, $\qquad r = -48$.

$c = -\frac{1}{9}$, $\qquad d = \frac{1}{27}$.

$A^3 = -d - \sqrt{d^2 + c^3} = -d = -\frac{1}{27} = R^3\{\cos 3\Theta + i\sin 3\Theta\}$,
$R = \frac{1}{3}$, $\qquad \Theta = 60°$.

1) $A = R\{\cos(\Theta + 0°) \quad + i\sin(\Theta + 0°) \quad \} = \frac{1}{6}\left(1 + \sqrt{3}\,i\right)$

2) $A = R\{\cos(\Theta + 120°) + i\sin(\Theta + 120°)\} = -\frac{1}{3}$

3) $A = R\{\cos(\Theta + 240°) + i\sin(\Theta + 240°)\} = \frac{1}{6}\left(1 - \sqrt{3}\,i\right)$

1) $x = -\frac{1}{3}p + A - \frac{c}{A} = 4$

2) $x = -\frac{1}{3}p + A - \frac{c}{A} = 3$

3) $x = -\frac{1}{3}p + A - \frac{c}{A} = 4$

Case 3c; $c \neq 0$ and $d^2 + c^3 < 0$:

Solve $x^3 - 6x^2 - x + 30 = 0$ for x.
Solution:

$p = -6$, $\qquad q = -1$, $\qquad r = 30$.

$c = -\frac{13}{3}$, $\qquad d = 6$.

$A^3 = -d - \sqrt{d^2 + c^3} = -6 - 6.735753141\,i = R^3\{\cos 3\Theta + i\sin 3\Theta\}$,
$R = 2.081665999$, $\qquad \Theta = 76.10211375°$.

ALTERNATE FORM OF THE CUBIC FORMULA – EXAMPLES

1) $A = R\{\cos(\Theta + 0°) + i\sin(\Theta + 0°)\} = 0.5 + 2.020725942\ i$
2) $A = R\{\cos(\Theta + 120°) + i\sin(\Theta + 120°)\} = -2 - 0.5773502692\ i$
3) $A = R\{\cos(\Theta + 240°) + i\sin(\Theta + 240°)\} = 1.5 - 1.443375673\ i$

1) $x = -\frac{1}{3}p + A - \frac{c}{A} = 3$

2) $x = -\frac{1}{3}p + A - \frac{c}{A} = -2$

3) $x = -\frac{1}{3}p + A - \frac{c}{A} = 5$