EXAMPLES

THE CUBIC FORMULA

These examples use the formulae presented in the "Roots of Polynomials" document, and which are derived in "The Cubic Formula" document.

Case 1: $D = 0$.

$x^3 - x^2 - 8x + 12 = 0,$

$p = -1, \quad q = -8, \quad r = 12 \qquad \Longrightarrow \qquad c = -\frac{25}{9}, \quad d = \frac{125}{27}, \quad D = 0.$

$\sqrt[3]{d} = \frac{5}{3}.$

So,

$x = -\frac{1}{3}p + \sqrt[3]{d} = 2 \qquad$ (twice),

$x = -\frac{1}{3}p - 2\sqrt[3]{d} = -3 \quad$ (once).

Case 2: $D > 0$.

$x^3 - 2x^2 + 10x + 136 = 0,$

$p = -2, \quad q = 10, \quad r = 136 \qquad \Longrightarrow \qquad c = \frac{26}{9}, \quad d = \frac{1918}{27}, \quad D = \frac{136{,}900}{27}.$

$\sqrt{D} = \frac{370\sqrt{3}}{9}.$

$A^3 = -d + \sqrt{D} = 0.1694961630, \qquad A = 0.5534180126.$

$B^3 = -d - \sqrt{D} = -142.2435702, \qquad B = -5.220084679.$

$x = -\frac{1}{3}p + A + B = -4,$

$x = -\frac{1}{3}p - \frac{1}{2}(A + B) \pm \frac{\sqrt{3}}{2}(A - B)i = 3 \pm 5i.$

Case 3: $D < 0$.

$x^3 - 5x^2 - 29x + 105 = 0,$

$p = -5, \quad q = -29, \quad r = 105 \quad \Longrightarrow \qquad c = -\frac{112}{9}, \quad d = \frac{640}{27}, \quad D = -\frac{4096}{3}.$

$\sqrt{-D} = \frac{64\sqrt{3}}{3}.$

$A^3 = -d + i\sqrt{-D} = -23.7037037 + 36.95041723i.$

$A = 2.666666667 + 2.309401077i,$

Note: you can take the cube root with your graphing calculator. Just enter $(-23.7037037 + 36.95041723i)$^$(1/3)$.

$\mathrm{Re}A = 2.666666667, \quad \mathrm{Im}A = 2.309401077.$

EXAMPLES

$x = -\frac{1}{3}p + 2\mathrm{Re}A = 7,$

$x = -\frac{1}{3}p - \mathrm{Re}A + \sqrt{3}\mathrm{Im}A = 3,$

$x = -\frac{1}{3}p - \mathrm{Re}A - \sqrt{3}\mathrm{Im}A = -5.$

THE QUARTIC FORMULA

These examples use the formulae presented in the "Roots of Polynomials" document, and which are derived in "The Quartic Formula" document.

Case 1: $e = 0$.

$x^4 + x^3 + \frac{9}{4}x^2 + x - \frac{21}{4} = 0,$

$p = 1, \quad q = \frac{9}{4}, \quad r = 1, \quad s = -\frac{21}{4} \qquad \Longrightarrow \qquad d = \frac{15}{8}, \; e = 0, \; f = -\frac{1375}{256}.$

$y^4 + dy^2 + f = 0 \qquad \Longrightarrow$ via the Quadratic Formula $\qquad y^2 = \frac{25}{16}$ and $y^2 = -\frac{55}{16}$ so that

$y = \pm\frac{5}{4}, \qquad y = \pm\frac{\sqrt{55}}{4}i.$

So,

$x = -\frac{1}{4}p + y \quad \Longrightarrow \qquad x = 1, \; x = -\frac{3}{2}, \; x = \frac{1}{4}\left(-1 \pm \sqrt{55}i\right).$

Case 2: $e \neq 0$.

$x^4 - 2x^3 - 30x^2 + 166x - 455 = 0,$

$p = -2, \quad q = -30, \quad r = 166, \quad s = -455 \quad \Longrightarrow \qquad d = -\frac{63}{2}, \quad e = 135, \quad f = -\frac{6075}{16}.$

$A^6 - 15.75A^4 + 156.9375A^2 - 284.765625 = 0.$

Note: This can be solved via the Cubic Formula, or, you could use you graphing calculator.

$A^2 = 2.25 \Longrightarrow$

$A = 1.5 = \frac{3}{2},$

$B^2 = 9, \qquad C^2 = -36, \qquad B = 3, \qquad C = 6i.$

So,

$x = -\frac{1}{4}p + A + iB = 2 + 3i,$

$x = -\frac{1}{4}p + A - iB = 2 - 3i,$

$x = -\frac{1}{4}p - A + iC = -7,$

$x = -\frac{1}{4}p - A - iC = 5.$