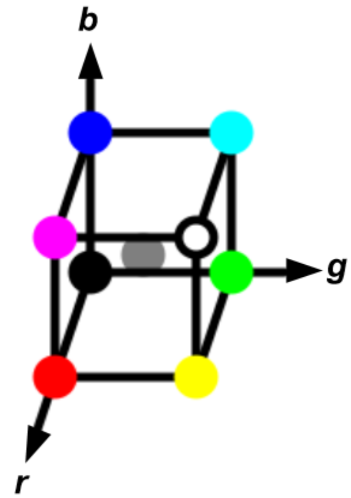
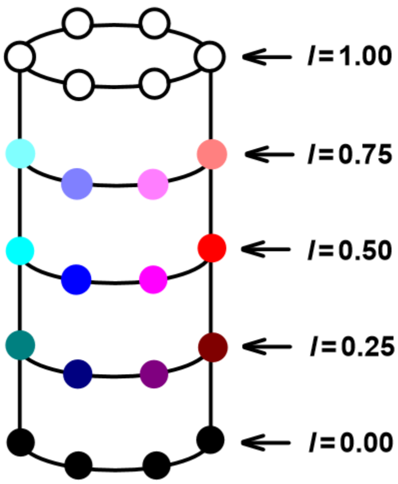
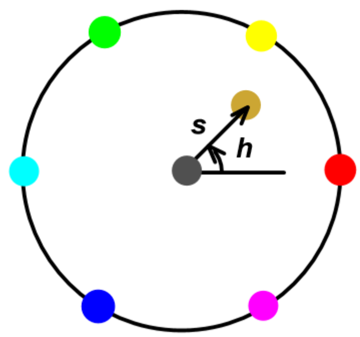
THE RED – GREEN – BLUE (RGB) AND HUE – SATURATION – LIGHTNESS (HSL) COLOR SPACES

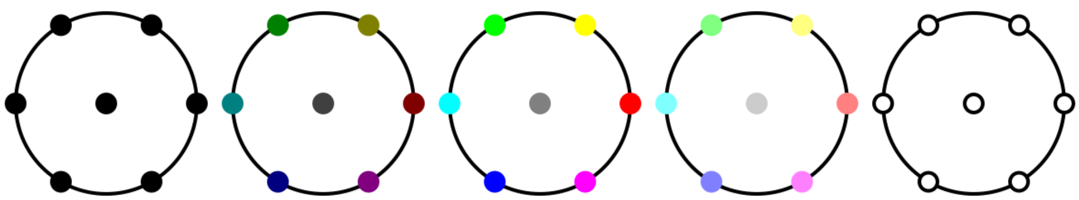
The figure at right shows the rectangular coordinates (r, g, b) of the RGB color space, where the coordinates have the ranges $r \in [0.0,1.0]$, $g \in [0.0,1.0]$ and $b \in [0.0,1.0]$. Colors other than red, green and blue follow from the light-addition of colors, *e.g.*, red + green = yellow, green + blue = cyan, blue + red = magenta and red + green + blue = white. The absence of light, *i.e.*, $(r, g, b) = (0.0,0.0,0.0)$, corresponds to black. Note that point at the centroid of the cube $(r, g, b) = (0.5,0.5,0.5)$ corresponds to grey.



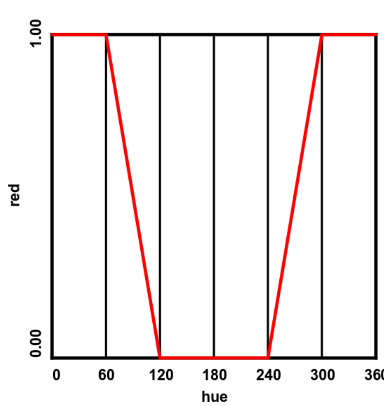
The HSL color space is defined by a cylindrical coordinate system with coordinates (h, s, l), whose ranges are $h \in [0,360)$, $s \in [0.0,1.0]$ and $l \in [0.0,1.0]$. The plane $l = 0.5$ of the coordinate system is shown directly at left, and the entire cylinder is shown at far left. In the plane $l = 0.5$, $s = 0$ corresponds to grey, *i.e.*, to $(r, g, b) = (0.5,0.5,0.5)$, and $s = 1$ corresponds to the fully saturated pure colors. Note that the view of the entire cylinder shown at left corresponds to $h \in [180,360)$. In any case, $l = 0$ (the bottom face) corresponds to black, and $l = 1$ (the top face) corresponds to white. Also, $s = 0$ and $l \in [0,1]$ gives the continuum of the greyscale colors. Finally, the five planes $l = 0.00$, 0.25 , 0.50 , 0.75 and 1.00 (respectively) are shown directly below. Note that a painter would call $l = 0.25$ a shade; and $l = 0.75$, a tint.

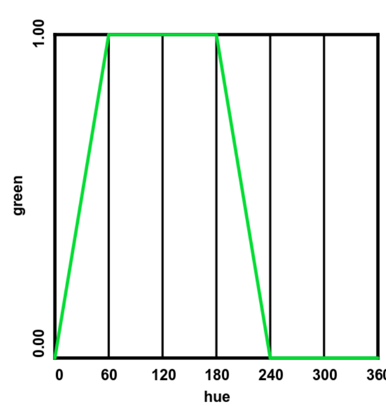


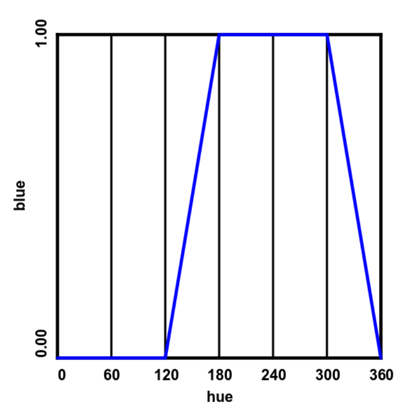




The fully saturated pure colors, *i.e.*, $l = 0.5$, $s = 1$ and $h \in [0,360)$, are obtained by linearly interpolating the $(r, g, b) \equiv (r_P, g_P, b_P)$ components as shown by the three graphs shown below.







Letting $H = h/60$, in equation form, these three graphs are

$$r_P(h) = \begin{Bmatrix} 1 & , & 0 \le h < 60 \\ -H+2 & , & 60 \le h < 120 \\ 0 & , & 120 \le h < 240 \\ H-4 & , & 240 \le h < 300 \\ 1 & , & 300 \le h < 360 \end{Bmatrix}, \quad (1a)$$

$$g_P(h) = \begin{Bmatrix} H & , & 0 \le h < 60 \\ 1 & , & 60 \le h < 180 \\ -H+4 & , & 180 \le h < 240 \\ 0 & , & 240 \le h < 360 \end{Bmatrix} \quad (1b)$$

and

$$b_P(h) = \begin{Bmatrix} 0 & , & 0 \le h < 120 \\ H-2 & , & 120 \le h < 180 \\ 1 & , & 180 \le h < 300 \\ -H+6 & , & 300 \le h < 360 \end{Bmatrix}. \quad (1c)$$

Now, denoting $(r, g, b) \equiv (r_B, g_B, b_B)$ as the colors residing in the (base) plane $l = 0.5$, we have, by linear interpolation

$$\begin{bmatrix} r_B(h,s) \\ g_B(h,s) \\ b_B(h,s) \end{bmatrix} = \begin{bmatrix} (r_P - 0.5)s + 0.5 \\ (g_P - 0.5)s + 0.5 \\ (b_P - 0.5)s + 0.5 \end{bmatrix}. \quad (2)$$

Finally, the RGB color, as a function of the HSL color, is, by linear interpolation,

$$r(h,s,l) = \begin{Bmatrix} 2r_B l & , & 0.0 \le l < 0.5 \\ 2(1-r_B)l + 2r_B - 1 & , & 0.5 \le l \le 1.0 \end{Bmatrix}, \quad (3a)$$

$$g(h,s,l) = \begin{Bmatrix} 2g_B l & , & 0.0 \le l < 0.5 \\ 2(1-g_B)l + 2g_B - 1 & , & 0.5 \le l \le 1.0 \end{Bmatrix} \text{ and} \quad (3b)$$

$$b(h,s,l) = \begin{Bmatrix} 2b_B l & , & 0.0 \le l < 0.5 \\ 2(1-b_B)l + 2b_B - 1 & , & 0.5 \le l \le 1.0 \end{Bmatrix}. \quad (3c)$$

Summary of the HSL Equations: Combining eqns. (1) through (3) above gives:

(i) $0 \le h \le 60$:

$$r(h,s,l) = \begin{Bmatrix} (1+s)l & , & 0.0 \le l < 0.5 \\ (1-s)l + s & , & 0.5 \le l \le 1.0 \end{Bmatrix} \quad (4a)$$

$$g(h,s,l) = \begin{Bmatrix} [\,(2H-1)\,s + 1\,]\,l & , & 0.0 \le l < 0.5 \\ [\,1 - (2H-1)\,s\,]\,l + (2H-1)s & , & 0.5 \le l \le 1.0 \end{Bmatrix} \quad (4b)$$

$$b(h,s,l) = \begin{Bmatrix} (1-s)l & , & 0.0 \le l < 0.5 \\ (1+s)l - s & , & 0.5 \le l \le 1.0 \end{Bmatrix} \quad (4c)$$

(ii) $60 < h \le 120$:

$$r(h,s,l) = \begin{Bmatrix} [\,(3-2H)\,s + 1]\,l & , & 0.0 \le l < 0.5 \\ [\,1 - (3-2H)\,s\,]\,l + (3-2H)s & , & 0.5 \le l \le 1.0 \end{Bmatrix} \quad (5a)$$

$$g(h,s,l) = \begin{Bmatrix} (1+s)l & , & 0.0 \le l < 0.5 \\ (1-s)l + s & , & 0.5 \le l \le 1.0 \end{Bmatrix} \quad (5b)$$

$$b(h,s,l) = \begin{Bmatrix} (1-s)l & , & 0.0 \le l < 0.5 \\ (1+s)l - s & , & 0.5 \le l \le 1.0 \end{Bmatrix} \quad (5c)$$

(iii) $120 < h \leq 180$:

$$r(h,s,l) = \begin{Bmatrix} (1-s)l & , & 0.0 \leq l < 0.5 \\ (1+s)l - s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{6a}$$

$$g(h,s,l) = \begin{Bmatrix} (1+s)l & , & 0.0 \leq l < 0.5 \\ (1-s)l + s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{6b}$$

$$b(h,s,l) = \begin{Bmatrix} [\,(2H-5)\,s+1]\,l & , & 0.0 \leq l < 0.5 \\ [\,1-(2H-5)\,s\,]\,l + (2H-5)s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{6c}$$

(iv) $180 < h \leq 240$:

$$r(h,s,l) = \begin{Bmatrix} (1-s)l & , & 0.0 \leq l < 0.5 \\ (1+s)l - s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{7a}$$

$$g(h,s,l) = \begin{Bmatrix} [\,(7-2H)\,s+1]\,l & , & 0.0 \leq l < 0.5 \\ [\,1-(7-2H)\,s\,]\,l + (7-2H)s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{7b}$$

$$b(h,s,l) = \begin{Bmatrix} (1+s)l & , & 0.0 \leq l < 0.5 \\ (1-s)l + s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{7c}$$

(v) $240 < h \leq 300$:

$$r(h,s,l) = \begin{Bmatrix} [\,(2H-9)\,s+1]\,l & , & 0.0 \leq l < 0.5 \\ [\,1-(2H-9)\,s\,]\,l + (2H-9)s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{8a}$$

$$g(h,s,l) = \begin{Bmatrix} (1-s)l & , & 0.0 \leq l < 0.5 \\ (1+s)l - s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{8b}$$

$$b(h,s,l) = \begin{Bmatrix} (1+s)l & , & 0.0 \leq l < 0.5 \\ (1-s)l + s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{8c}$$

(vi) $300 < h < 360$:

$$r(h,s,l) = \begin{Bmatrix} (1+s)l & , & 0.0 \leq l < 0.5 \\ (1-s)l + s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{9a}$$

$$g(h,s,l) = \begin{Bmatrix} (1-s)l & , & 0.0 \leq l < 0.5 \\ (1+s)l - s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{9b}$$

$$b(h,s,l) = \begin{Bmatrix} [\,(11-2H)\,s+1]\,l & , & 0.0 \leq l < 0.5 \\ [\,1-(11-2H)\,s\,]\,l + (11-2H)s & , & 0.5 \leq l \leq 1.0 \end{Bmatrix} \tag{9c}$$

Summary of the RGB Equations:

Equations (10) through (16) below give (h,s,l) as a function of (r,g,b).

Greyscale Cases: $r = g = b$.

$$\begin{aligned} (r,g,b) &= (0,0,0) &&\Rightarrow && l = 0 \quad (h \text{ and } s \text{ are immaterial}) \\ (r,g,b) &= (1,1,1) &&\Rightarrow && l = 1 \quad (h \text{ and } s \text{ are immaterial}) \\ (r,g,b) &= (x,x,x)\,, \quad x \in (0,1) &&\Rightarrow && (s,l) = (0,x) \quad h \text{ is immaterial} \end{aligned} \tag{10}$$

Non-greyscale Cases: Recall that $h = 60H$.

(i) r is max. b is min. (or r is max. $g = b$ is min.) (or $r = g$ is max. b is min.) $\Rightarrow 0 \leq h \leq 60$:
Inversion of eqns. (4) gives

$$l = \frac{r+b}{2}\,, \quad s = \frac{r-b}{1-|2l-1|}\,, \quad H = \frac{g-b}{r-b}\,. \tag{11}$$

(ii) g is max. b is min. (or g is max. $r = b$ is min.) $\Rightarrow 60 < h \leq 120$:
Inversion of eqns. (5) gives

$$l = \frac{g + b}{2}\ , \quad s = \frac{g - b}{1 - |2l - 1|}\ , \quad H = \frac{-r + 2g - b}{g - b}\ . \tag{12}$$

(iii) g is max. r is min. (or $g = b$ is max. r is min.) $\Rightarrow 120 < h \leq 180$:
Inversion of eqns. (6) gives

$$l = \frac{g + r}{2}\ , \quad s = \frac{g - r}{1 - |2l - 1|}\ , \quad H = \frac{-3r + 2g + b}{g - r}\ . \tag{13}$$

(iv) b is max. r is min. (or b is max. $r = g$ is min.) $\Rightarrow 180 < h \leq 240$:
Inversion of eqns. (7) gives

$$l = \frac{b + r}{2}\ , \quad s = \frac{b - r}{1 - |2l - 1|}\ , \quad H = \frac{-3r - g + 4b}{b - r}\ . \tag{14}$$

(v) b is max. g is min. (or $r = b$ is max. g is min.) $\Rightarrow 240 < h \leq 300$:
Inversion of eqns. (8) gives

$$l = \frac{b + g}{2}\ , \quad s = \frac{b - g}{1 - |2l - 1|}\ , \quad H = \frac{r - 5g + 4b}{b - g}\ . \tag{15}$$

(vi) r is max. g is min. $\Rightarrow 300 < h < 360$:
Inversion of eqns. (9) gives

$$l = \frac{r + g}{2}\ , \quad s = \frac{r - g}{1 - |2l - 1|}\ , \quad H = \frac{6r - 5g - b}{r - g}\ . \tag{16}$$