1A.3. Equations of Parabolas

A Parabola is also called a Quadratic

$y = ax^2 + bx + c$ ← Standard Form

Example #1. Find the equation of the parabola which passes through the three points $(-2,3)$, $(0,0)$ and $(4,0)$.

SOLUTION: $y = ax^2 + bx + c$

$(-2,3) \rightarrow 3 = a(-2)^2 + b(-2) + c \rightarrow 4a - 2b + c = 3 \rightarrow 4a - 2b = 3$

$(0,0) \rightarrow 0 = a(0)^2 + b(0) + c \rightarrow c = 0 \rightarrow 16a + 4b = 0$

$(4,0) \rightarrow 0 = a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = 0$

$-4(4a - 2b = 3)$ $\quad -16a + 8b = -12$

$16a + 4b = 0$

$12b = -12 \quad b = -1$

$4a - 2b = 3 \Rightarrow$

$4a - 2(-1) = 3$

$4a + 2 = 3, \quad 4a = 1, \quad a = \frac{1}{4}$

$y = \frac{1}{4}x^2 - 1 \cdot x + 0, \quad y = \frac{1}{4}x^2 - x$ ◄

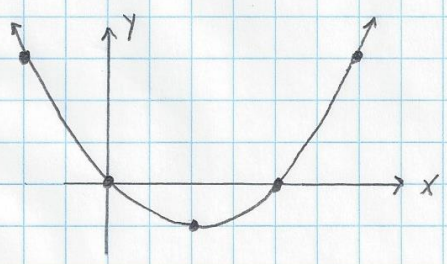
Example #2. Verify the result of Example #1 using quadratic regression.

SOLUTION: $y = 0.25x^2 - 1 \cdot x - (2.3 \times 10^{-12}), \quad y = \frac{1}{4}x^2 - x + 0, \quad y = \frac{1}{4}x^2 - x$ ◄

Note that $r^2 = 1$ (perfect correlation)

A Property of Parabolas

Example #3. The parabola $y = \frac{1}{4}x^2 - x$ is graphed below.
Graph the slope of the parabola.



SOLUTION:

x	y
-2	3
0	0
2	-1
4	0
6	3

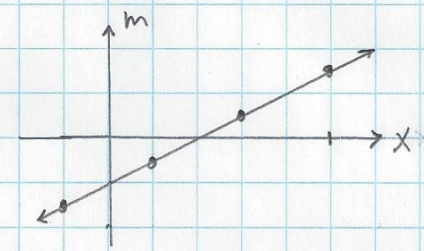
1A.3. Equations of Parabolas

$$m(-1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{0-(-2)} = \frac{-3}{2} = -1.5$$

-1: half way between -2 and 0

$$m(1) = \frac{-1-0}{2-0} = \frac{-1}{2} = -0.5, \quad m(3) = \frac{0-(-1)}{4-2} = \frac{1}{2} = 0.5, \quad m(5) = \frac{3-0}{6-4} = \frac{3}{2} = 1.5$$

x	m
−1	−1.5
1	−0.5
3	0.5
5	1.5



Note that the equation for the slope of the parabola is

$$m = \frac{1}{2}x - 1$$

FACT The slope of a parabola is a line.