1A.4. The Quadratic Formula

We want to solve the quadratic equation $ax^2+bx+c=0$ for x.

$x^2+\frac{b}{a}x+\frac{c}{a}=0$, $x^2+\frac{b}{a}x=-\frac{c}{a}$ (1)

Look at $\left(x+\frac{b}{2a}\right)^2=\left(x+\frac{b}{2a}\right)\left(x+\frac{b}{2a}\right)=x^2+\frac{b}{2a}x+\frac{b}{2a}x+\frac{b^2}{4a^2}=x^2+\frac{b}{a}x+\frac{b^2}{4a^2}$,

$\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}=x^2+\frac{b}{a}x$ (2). Put eqn. (2) into eqn. (1) $\Rightarrow$

$\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}=-\frac{c}{a}$, $\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}\cdot\frac{4a}{4a}=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$,

$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$, $x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$, $x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Quadratic Formula

Examples. Solve the indicated quadratic equation with the Quadratic Formula.

Example #1. $x^2-5x-24=0$

SOLUTION: $a=1$, $b=-5$, $c=-24$, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{5\pm\sqrt{(-5)^2-4(1)(-24)}}{2(1)}=$

$=\frac{5\pm\sqrt{121}}{2}=\frac{5\pm11}{2}$ $\quad x=\frac{5-11}{2}=\frac{-6}{2}=-3$ ◀

$x=\frac{5+11}{2}=\frac{16}{2}=8$ ◀

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Example #2. $10x^2 - x - 24 = 0$

SOLUTION: $a = 10$, $b = -1$, $c = -24$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{1 \pm \sqrt{(-1)^2 - 4(10)(-24)}}{2(10)}$

$= \dfrac{1 \pm \sqrt{961}}{20} = \dfrac{1 \pm 31}{20}$

$x = \dfrac{1-31}{20} = \dfrac{-30}{20} = -1.5 = -\dfrac{3}{2}$ ◀

$x = \dfrac{1+31}{20} = \dfrac{32}{20} = 1.6 = \dfrac{8}{5}$ ◀