

1A.6. Exponential Equations

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$$y = a \cdot b^x$$

$a \equiv$ initial value, i.e., y at $x=0$
 $b \equiv$ base ($b > 0$)

Example #1. Find the exponential equation which passes through the two points $(0, 3.000)$ and $(3, 10.125)$.

SOLUTION:

$$\begin{aligned} y &= a \cdot b^x & (0, 3) &\rightarrow 3 = a \cdot b^0 = a, & y &= 3 \cdot b^x \\ (3, 10.125) &\rightarrow 10.125 = 3 \cdot b^3, & \frac{10.125}{3} &= b^3, & 3.375 &= b^3, \\ b &= \sqrt[3]{3.375} = 1.5, & y &= 3(1.5)^x \end{aligned}$$

Example #2. Find the exponential equation which passes through the two points $(2, 10.8)$ and $(4, 97.2)$.

SOLUTION:

$$\begin{aligned} y &= a \cdot b^x & (4, 97.2) &\rightarrow 97.2 = a \cdot b^4 \\ (2, 10.8) &\rightarrow 10.8 = a \cdot b^2 & (\div) &\rightarrow 9 = b^2, & b &= \sqrt{9} = 3 \\ 10.8 &= a \cdot 3^2, & a &= 1.2 & y &= 1.2(3)^x \end{aligned}$$

Population Growth

Example #3. In 2000, the population of Atlanta, GA was 0.419 million people. In 2020 it was 0.499 million. Estimate the population of Atlanta in the year 2050.

SOLUTION:

$$\begin{aligned} 2000 &\rightarrow t=0 & p &= a \cdot b^t & p &\equiv \text{population} \\ 2020 &\rightarrow t=20 \\ 2050 &\rightarrow t=50 & (0, 0.419) &\rightarrow 0.419 = a \cdot b^0 = a \end{aligned}$$

$$p = 0.419 \cdot b^t \quad (20, 0.499) \rightarrow 0.499 = 0.419 \cdot b^{20}$$

$$\frac{0.499}{0.419} = b^{20}, \quad b = \sqrt[20]{\frac{0.499}{0.419}} = 1.008775036$$

$$p = 0.419 \cdot b^{50} = 0.649 \text{ million}$$

The population of Atlanta increases by 0.8775% per year

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Radioactive Decay

Example #4. In 5730 years, half of a sample of Carbon-14 (C-14) will decay into (stable) Carbon-12 (C-12). The sample of C-14 is initially 10 grams. After 7500 years, how much of the sample will have decayed?

SOLUTION: $g \equiv$ grams of C-12, $g = a \cdot b^t$, $g = 10 \cdot b^t$.

$$(5730, 5) \quad , \quad 5 = 10 \cdot b^{5730} \quad , \quad 0.5 = b^{5730} \quad ,$$

$$b = \sqrt[5730]{0.5} = 0.9998790392$$

$1-b = 0.00121\%$
of the sample
decays each year

$$g = 10 \cdot b^{7500} = 4.036 \text{ g C-12 left} \Rightarrow 10 - 4.036 = 5.964 \text{ g have decayed}$$

Example #5. Do Example #2 using exponential regression.

SOLUTION:

$$y = 1.2(3)^x \quad \leftarrow \text{Note that } r^2 = 1 \text{ (perfect correlation)}$$