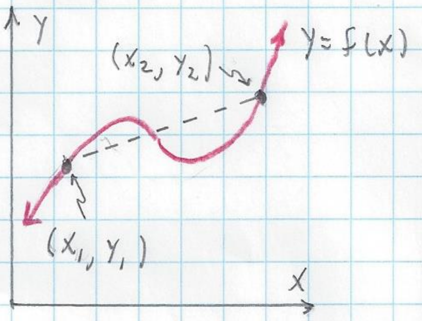
18.1. Average Rate of Change



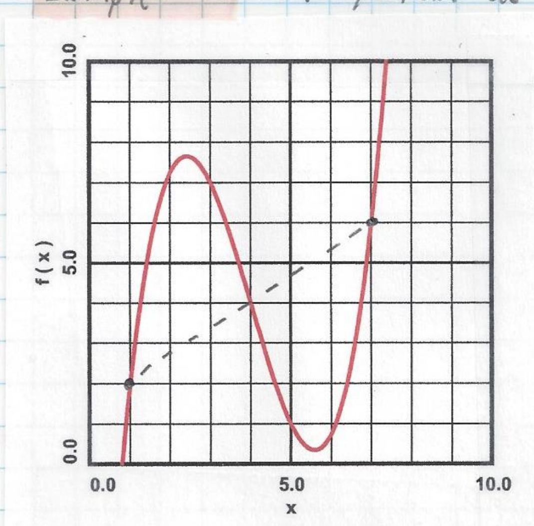
$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$ ← slope formula

$m \equiv$ the average rate of change of f with respect to x on $x_1 \le x \le x_2$
$\equiv$ the average slope of f on $x_1 \le x \le x_2$

Example #1. Calculate the average rate of change of $f(x) = x^2 - 6x + 11$ over $1 \le x \le 7$.

SOLUTION: $m = \dfrac{f(7) - f(1)}{7-1} = \dfrac{18 - 6}{7-1} = \dfrac{12}{6} = 2$

Example #2. For $y = f(x)$ as defined by the graph, calculate the average rate of change of f with respect to x on $1 \le x \le 7$.



SOLUTION:

$m = \dfrac{f(7) - f(1)}{7-1} = \dfrac{6 - 2}{7-1} = \dfrac{4}{6} = \dfrac{2}{3}$

Example #3. The table shows the distance d (in feet) travelled by a downhill skier in t seconds. Calculate the average rate of change of d with respect to t on

t	d
0	0.0
2	8.3
4	33.3
6	75.0
8	133.3
10	208.3
12	300.0

(a) $0 \le t \le 4$ sec (b) $4 \le t \le 8$ sec (c) $8 \le t \le 12$ sec

(d) What is the physical meaning of the rate of change?

SOLUTION:

(a) $\dfrac{\Delta d}{\Delta t} = \dfrac{33.3 - 0.0}{4 - 0} = 8.33 \dfrac{ft}{sec}$ ◄

(b) $\dfrac{\Delta d}{\Delta t} = \dfrac{133.3 - 33.3}{8 - 4} = 25.00 \dfrac{ft}{sec}$ ◄

(c) $\dfrac{\Delta d}{\Delta t} = \dfrac{300.0 - 133.3}{12 - 8} = 41.7 \dfrac{ft}{sec}$ ◄

(d) $\dfrac{\Delta d}{\Delta t}$ is the speed of the skier ◄