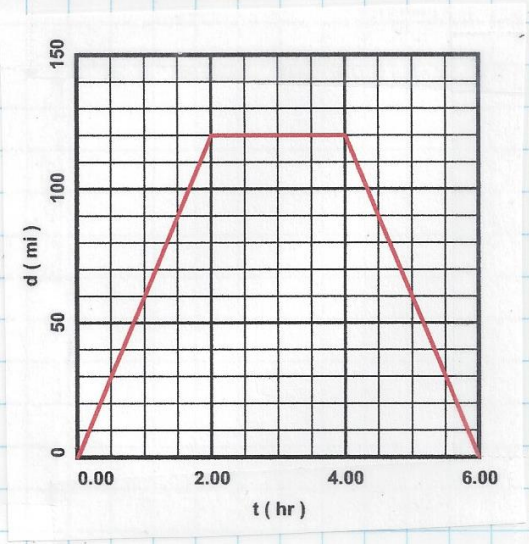
1B.3. More Rates of Change

Example #1. Steve's cousin lives 120 miles from his house, and he decides to drive to visit her. The graph shows Steve's distance d, in miles, from his house as a function of time t, in hours.



(a) Calculate the average rate of change of d with respect to t for the intervals

(1) $0 \le t \le 2$ hrs
(2) $2 \le t \le 4$ hrs
(3) $4 \le t \le 6$ hrs.

What is the meaning of each of these rates of change?

(b) Also calculate the average rate of change for

(4) $0 \le t \le 4$ hrs (5) $0 \le t \le 6$ hrs.

SOLUTION:

(1) $\frac{\Delta d}{\Delta t} = \frac{120-0}{2-0} = 60 \frac{mi}{hr}$ ◀ Steve drives for 2 hours at 60 mph to his cousin's house ◀

(2) $\frac{\Delta d}{\Delta t} = \frac{120-120}{4-2} = 0 \frac{mi}{hr}$ ◀ Steve is not moving (he is at his cousin's house for 2 hours) ◀

(3) $\frac{\Delta d}{\Delta t} = \frac{0-120}{6-4} = -60 \frac{mi}{hr}$ ◀ Steve is driving back home at 60 mph from his cousin's house to his house ◀

(4) $\frac{\Delta d}{\Delta t} = \frac{120-0}{4-0} = 30 \frac{mi}{hr}$ ◀

(5) $\frac{\Delta d}{\Delta t} = \frac{0-0}{6-0} = 0 \frac{mi}{hr}$ ◀

Example #2. In 1960, a McDonald's hamberger costed \$0.21. In 2020, the cost of a hamburger was \$6.19.

(a) Calculate the average rate of change of the cost of a hamberger for the time interval $1960 \le t \le 2020$ years.

(b) Assuming that the rate of change is always valid

(i) Calculate the cost of a hamberger in the year 2030.

(ii) In what year will the hamberger cost \$10.00?

SOLUTION:

(a) $h \equiv$ cost of hamberger, $t \equiv$ time (the year)

$$\frac{\Delta h}{\Delta t} = \frac{6.19 - 0.21}{2020 - 1960} = \frac{5.98}{60} = 0.09\overline{96} \ \frac{\text{dollars}}{\text{year}}$$

(b) $h = 0.09\overline{96}\,t + b$, $0.21 = 0.09\overline{96}\,(1960) + b$, $b = -195.13\overline{6}$

$$h = 0.09\overline{96}\,t - 195.13\overline{6}$$

(i) $h = 0.09\overline{96}\,(2030) - 195.13\overline{6} = \$7.19$ ◀

(ii) $10 = 0.09\overline{96}\,t - 195.13\overline{6}$, $0.09\overline{96}\,t = 205.13\overline{6}$, $t = 2058$ ◀