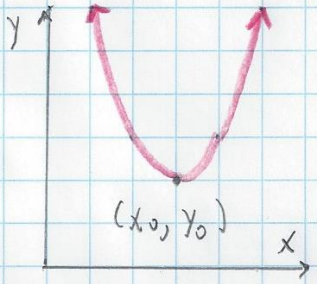
1B.5. Completing the Square

Completing the Square

$$\left(x+\frac{b}{2}\right)^2 = \left(x+\frac{b}{2}\right)\left(x+\frac{b}{2}\right) = x^2+\frac{b}{2}x+\frac{b}{2}x+\frac{b^2}{4} = x^2+bx+\frac{b^2}{4}$$

Take half $= \frac{b}{2}$ → Then square $= \frac{b^2}{4}$

Vertex Form of a Parabola

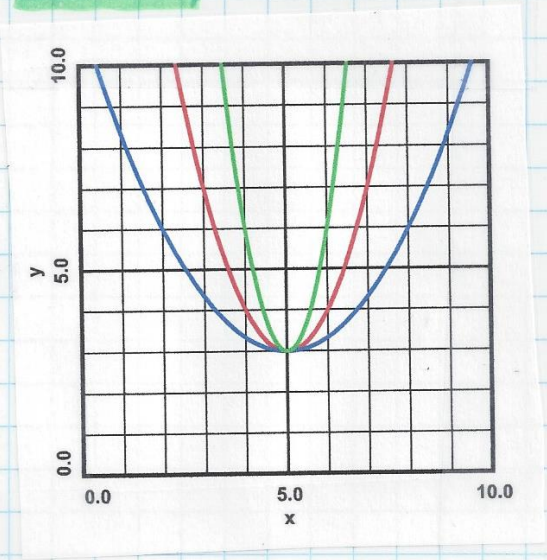


$(x_0, y_0) \equiv$ vertex

$$y - y_0 = a(x-x_0)^2$$

Example #1. Graph $y-3=(x-5)^2$, $y-3=3(x-5)^2$ and $y-3=\frac{1}{3}(x-5)^2$.

SOLUTION:



$y-3=(x-5)^2$

$y-3=3(x-5)^2$

$y-3=\frac{1}{3}(x-5)^2$

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Example #2. Convert the parabola in standard form to vertex form by completing the square.

(a) $y = x^2 - 6x + 5$ (b) $y = -3x^2 - 12x - 4$

SOLUTION:

(a) $y - 5 = x^2 - 6x$, $y + 4 = (x^2 - 6x + 9)$, $y + 4 = (x-3)^2$ ◀
(+9) (+9)

(b) $y + 4 = -3x^2 - 12x$, $y + 4 = -3(x^2 + 4x)$, $y - 8 = -3(x^2 + 4x + 4)$
(−3·4) (+4) $y - 8 = -3(x+2)^2$ ◀

Example #3. Convert the parabola $y - 11 = 4(x+8)^2$ to standard form.

SOLUTION:

$y - 11 = 4(x^2 + 16x + 64)$, $y - 11 = 4x^2 + 64x + 256$, $y = 4x^2 + 64x + 267$ ◀

Example #4. Solve the quadratic equations for x by completing the square.

(a) $x^2 - 6x - 16 = 0$ (b) $8x^2 - 2x - 15 = 0$

SOLUTION:

(a) $x^2 - 6x = 16$, $x^2 - 6x + 9 = 25$, $(x-3)^2 = 25$,
(+9) (+9)

$x - 3 = \pm 5$, $x = 3 \pm 5$, $x = 8$ ◀ $x = -2$ ◀

(b) $8x^2 - 2x = 15$, $8(x^2 - \frac{1}{4}x) = 15$, $8(x^2 - \frac{1}{4}x + \frac{1}{64}) = \frac{121}{8}$,
($+\frac{1}{64}$) ($+\frac{8}{64}$)

$8(x - \frac{1}{8})^2 = \frac{121}{8}$, $(x - \frac{1}{8})^2 = \frac{121}{64}$, $x - \frac{1}{8} = \pm\frac{11}{8}$,

$x = \frac{1}{8} \pm \frac{11}{8}$, $x = \frac{12}{8} = \frac{3}{2}$ ◀, $x = -\frac{10}{8} = -\frac{5}{4}$ ◀