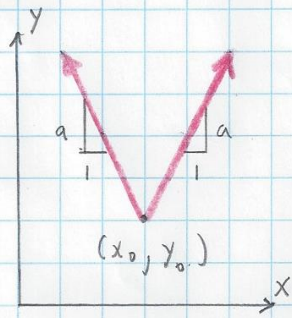
1B.6. Absolute Value Functions

Absolute Value

$|-5| = 5$, $|2| = 2$, etc.

Absolute Value Function

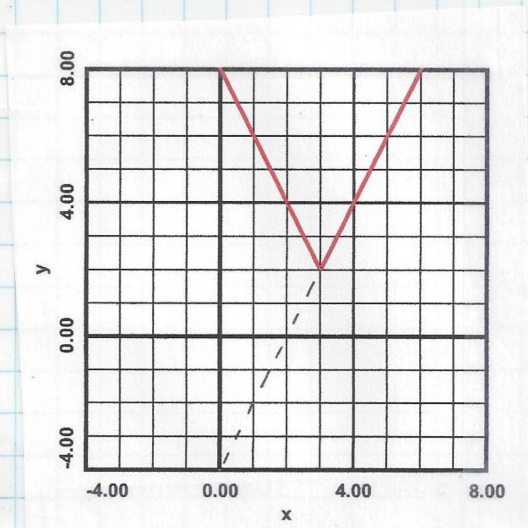


$(x_0, y_0) \equiv$ vertex

$$\boxed{y - y_0 = a\,|x - x_0|}$$

Example #1. Graph $y - 2 = 2|x - 3|$

SOLUTION:



Example #2. By looking at the graph in Example #1, convert $y - 2 = 2|x - 3|$ to a piecewise-defined function containing no absolute values.

SOLUTION:

$$y = \begin{cases} -2x + 8, & x \le 3 \\ 2x - 4, & x > 3 \end{cases}$$

Example #3. Algebraically convert $y + 3 = 3|x - 1|$ to a piecewise-defined function containing no absolute values.

SOLUTION: vertex is at $(1, -3)$

$x \le 1$: $y + 3 = -3(x - 1)$, $y + 3 = -3x + 3$, $y = -3x$

$x > 1$: $y + 3 = 3(x - 1)$, $y + 3 = 3x - 3$, $y = 3x - 6$

$$y = \begin{cases} -3x, & x \le 1 \\ 3x - 6, & x > 1 \end{cases}$$

Example #4. For the function in Example #3, calculate the average rate of change of y with respect to x on the interval $-1 \le x \le 4$

SOLUTION:

$Y(4) = 3(4) - 6 = 6$, $Y(-1) = -3(-1) = 3$, $\dfrac{Y(4) - Y(-1)}{4 - -1} = \dfrac{6-3}{4+1} = \dfrac{3}{5}$

Example #5. Algebraically, convert the piecewise-defined function

$$y = \begin{cases} -\frac{1}{2}x + 5 & , x \le 4 \\ \frac{1}{2}x + 1 & , x > 4 \end{cases}$$

to an absolute value function.

SOLUTION: $x = 4$ is the vertex, $y = \begin{cases} -\frac{1}{2}(4) + 5 = 3 \\ \frac{1}{2}(4) + 1 = 3 \end{cases} = 3 \Rightarrow$

$y - 3 = \frac{1}{2}|x - 4|$