2A.4. Translation of Functions

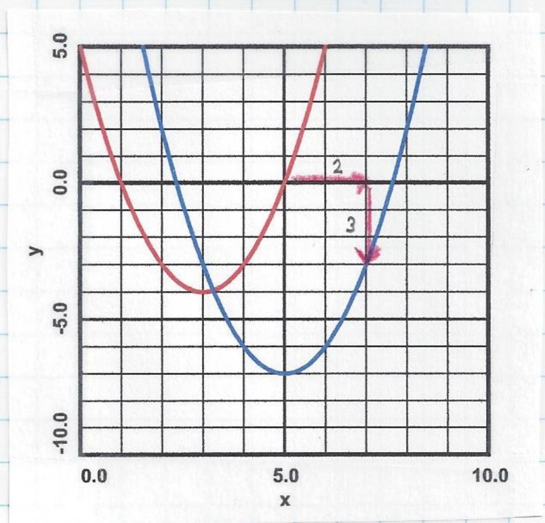
Translation $\equiv$ movement with no deformation and no rotation.

FACT: The graph of $\boxed{y - y_0 = f(x - x_0)}$ is a translation, by x_0 in the x-direction and y_0 in the y-direction, of the graph of $\boxed{y = f(x)}$

Example #1. For $y = f(x) = x^2 - 6x + 5$, construct $y = g(x)$ by translating $y = f(x)$ by $(x_0, y_0) = (2, -3)$, i.e., calculate $g(x) = f(x - x_0) + y_0$.
Also, graph both $y = f(x)$ and $y = g(x)$ to verify the translation.

SOLUTION:

$g(x) = f(x-2) - 3 = (x-2)^2 - 6(x-2) + 5 - 3 = (x^2 - 4x + 4) + (-6x + 12) + 5 - 3 =$
$= x^2 - 10x + 13$



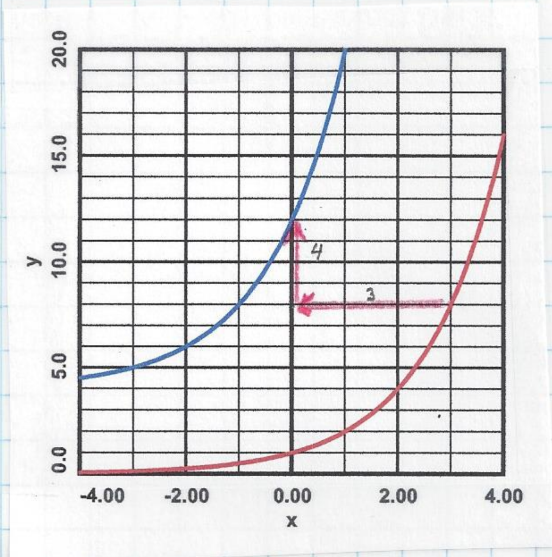
$y = f(x) = x^2 - 6x + 5$

$y = g(x) = x^2 - 10x + 13$

Example #2. For $y = f(x) = 2^x$, construct $y = g(x)$ by translating $y = f(x)$ by $(x_0, y_0) = (-3, 4)$, i.e., calculate $g(x) = f(x - x_0) + y_0$. Also, graph both $y = f(x)$ and $y = g(x)$ to verify the translation.

SOLUTION:

$g(x) = f(x+3) + 4 = 2^{x+3} + 4 = 2^x \cdot 2^3 + 4 = 8 \cdot 2^x + 4$ ◁



$y = f(x) = 2^x$

$y = g(x) = 8 \cdot 2^x + 4$