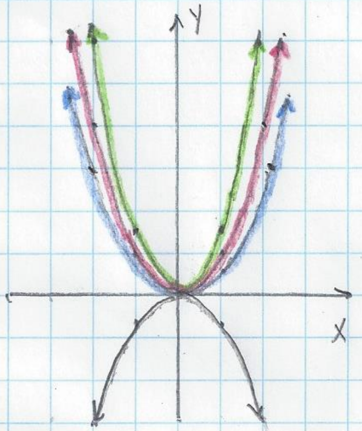
2A.5. Vertical Stretch and Compression

Consider



- $y = x^2$ (the parent function)
- $y = 1.5x^2$ (a vertical stretch of the parent function by a factor of 1.5)
- $y = 0.75x^2$ (a vertical compression of the parent function by a factor of 0.75)
- $y = -0.75x^2$ (not that the minus signs flips the function over the x-axis)

Facts

(1) $\boxed{y = af(x)}$ is a vertical stretch $(a > 1)$, or a vertical compression $(0 < a < 1)$, of $\boxed{y = f(x)}$ by a factor of a.

(2) $\boxed{y - y_0 = af(x - x_0)}$ is a translation, by (x_0, y_0), of $\boxed{y = af(x)}$

Comment ...

$y = x^2$ → vertical stretch or compression → $y = ax^2$ → translation → $y - y_0 = a(x - x_0)^2$

parent function

vertex form of a parabola

Example #1. Given that $y=f(x)=x^2-6x+11$.

(a) Graph $y=f(x)$.

(b) Vertically stretch $y=f(x)$ by a factor of 2 to obtain $y=g(x)$.

(c) Graph $y=g(x)$.

(d) Translate $y=g(x)$ by $(4,-3)$ to obtain $y=h(x)$.

(e) Graph $y=h(x)$

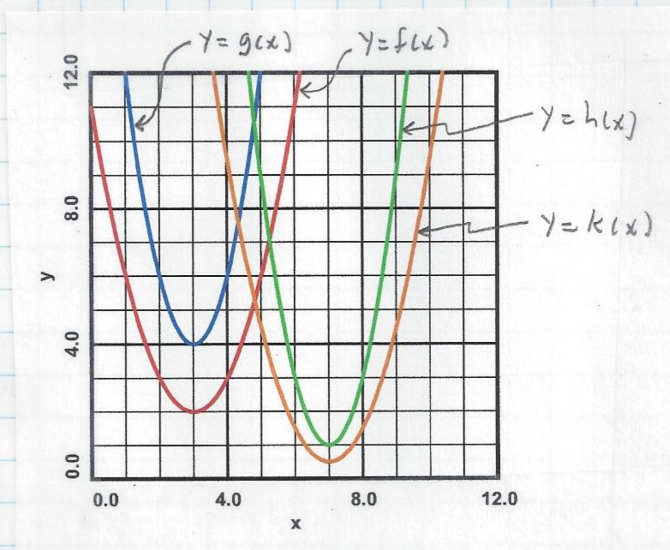
SOLUTION:

(b) $y=g(x)=2f(x)=2(x^2-6x+11)=2x^2-12x+22$ ◄

(d) $y=h(x)=g(x-4)-3= 2(x-4)^2-12(x-4)+22-3=$

$=2(x^2-8x+16)-12(x-4)+22-3=$

$=(2x^2-16x+32)+(-12x+48)+22-3=2x^2-28x+99$ ◄



Example #2. Show that the transformation in Example #1 of f(x) to h(x) is equivalent to:

(a) Translate $y=f(x)$ by $(4,-1.5)$ to obtain $y=k(x)$.

(b) Vertically stretch $y=k(x)$ by a factor of 2 to obtain $y=h(x)$.

(c) Also, graph $y=k(x)$ on the grid in Example #1.

SOLUTION:

(a) $k(x)=f(x-4)-1.5 = (x-4)^2-6(x-4)+11-1.5=$

$=(x^2-8x+16)+(-6x+24)+11-1.5= x^2-14x+49.5$ ◄

(b) $h(x)=2k(x)=2(x^2-14x+49.5)=2x^2-28x+99$ ◄

(same as in Example #1).