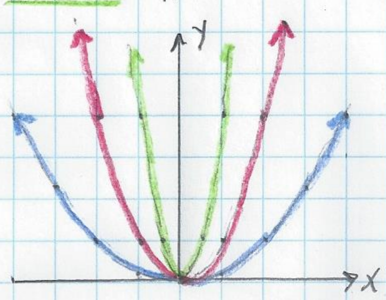
2A.6. Horizontal Stretch and Compression

Consider ...



— $y=x^2$ (parent function)

— $y=\left(\frac{1}{2}x\right)^2$ (a horizontal stretch, by a factor of 2, of the parent function)

— $y=(2x)^2$ (a horizontal compression, by a factor of $\frac{1}{2}$, of the parent function)

Facts $(a>1)$

(1) $\boxed{y=f\left(\frac{1}{a}x\right)}$ is a horizontal stretch, by a factor of a of the parent function $\boxed{y=f(x)}$.

(2) $\boxed{y=f(ax)}$ is a horizontal compression, by a factor of $\frac{1}{a}$, of the parent function $\boxed{y=f(x)}$.

Example #1. For $y=f(x)=\frac{1}{3}x^2-6x+29$,

(a) Graph $y=f(x)$.

(b) Construct a function $y=g(x)$ which is a horizontal compression, by a factor of $\frac{1}{3}$, of $y=f(x)$.

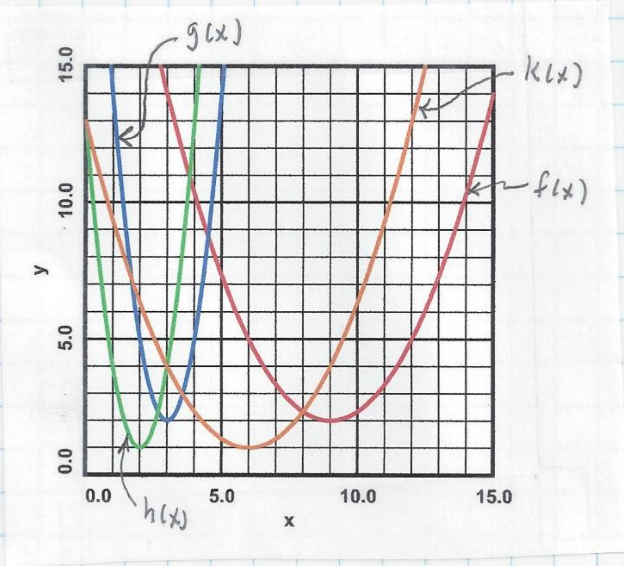
(c) Graph $y=g(x)$.

(d) Construct a function $y=h(x)$, which is a translation, by $(-1,-1)$, of $y=g(x)$

(e) Graph $y=h(x)$

SOLUTION!

2A.6. Horizontal Stretch and Compression



(b) $g(x) = f(3x) = \frac{1}{3}(3x)^2 - 6(3x) + 29 =$

$= \frac{1}{3}(9x^2) - 18x + 29$

$= 3x^2 - 18x + 29$

(d) $h(x) = g(x+1) - 1 =$

$= 3(x+1)^2 - 18(x+1) + 29 - 1 =$

$= 3(x^2 + 2x + 1) - 18(x+1) + 29 - 1 =$

$= (3x^2 + 6x + 3) + (-18x - 18) + 29 - 1 =$

$= 3x^2 - 12x + 13$

Example #2. The transformation of $y = f(x)$ to $y = h(x)$ in Example #1 is equivalent to:

(a) Construct a function $y = k(x)$, which is a translation, by $(-3, -1)$, of $y = f(x)$.

(b) Graph $y = k(x)$ on the axes in Example #1.

(c) Construct $y = h(x)$ which is a horizontal compression, by a factor of $\frac{1}{3}$, of $y = k(x)$.

SOLUTION:

(a) $k(x) = f(x+3) - 1 = \frac{1}{3}(x+3)^2 - 6(x+3) + 29 - 1 =$

$= \frac{1}{3}(x^2 + 6x + 9) - 6(x+3) + 29 - 1 = (\frac{1}{3}x^2 + 2x + 3) + (-6x - 18) + 29 - 1 =$

$= \frac{1}{3}x^2 - 4x + 13$

(c) $y = h(x) = k(3x) = \frac{1}{3}(3x)^2 - 4(3x) + 13 = \frac{1}{3}(9x^2) - 12x + 13 =$

$= 3x^2 - 12x + 13$ which is the same as in Example #1.