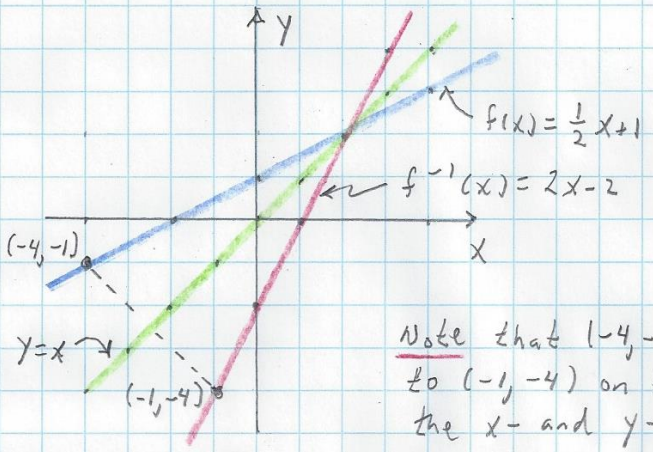
2B.1. Introduction to Inverse Functions

Inverse Functions

The graphs of inverse functions are reflectively symmetric about the line $y=x$.



$f(x) \equiv$ f of x

$f^{-1}(x) \equiv$ f inverse of x

Note that $(-4,-1)$ on $f(x)$ corresponds to $(-1,-4)$ on $f^{-1}(x)$. In other words, the x- and y-coordinates are switched.

Calculation of Inverse Functions

Example #1. For $f(x)=\frac{1}{2}x+1$, calculate $f^{-1}(x)$.

Solution:

$y=f(x)=\frac{1}{2}x+1$. Switch x and y $\Rightarrow$ $x=\frac{1}{2}y+1$. Now, solve for y...

$\frac{1}{2}y=x-1$, $y=2(x-1)$, $y=f^{-1}(x)=2x-2$

Inverse Function Properties

Inverse functions "undo" each other, in other words...

$$f(f^{-1}(x))=x, \quad f^{-1}(f(x))=x$$

Example #2. For $f(x)=\frac{1}{2}x+1$ and $f^{-1}(x)=2x-2$, calculate

(a) $f(f^{-1}(x))$ (b) $f^{-1}(f(x))$

SOLUTION:

(a) $f(f^{-1}(x))=f(f^{-1})=\frac{1}{2}f^{-1}+1=\frac{1}{2}(2x-2)+1=x-1+1=x$

(b) $f^{-1}(f(x))=f^{-1}(f)=2f-2=2\left(\frac{1}{2}x+1\right)-2=x+2-2=x$