

2B.2. Parabolas

1 of 2

Example #1. For $y = f(x) = -x^2 + 12x - 27$

(a) Calculate $y = f^{-1}(x)$

(b) Graph $y = f(x)$, $y = f^{-1}(x)$ and $y = x$ on the same set of axes.

Solution:

(a) Put $y = f(x)$ in vertex form... $y + 27 = -x^2 + 12x$,

$$y + 27 = -(x^2 - 12x) \quad , \quad y - 9 = -(x^2 - 12x + 36) = -(x - 6)^2$$

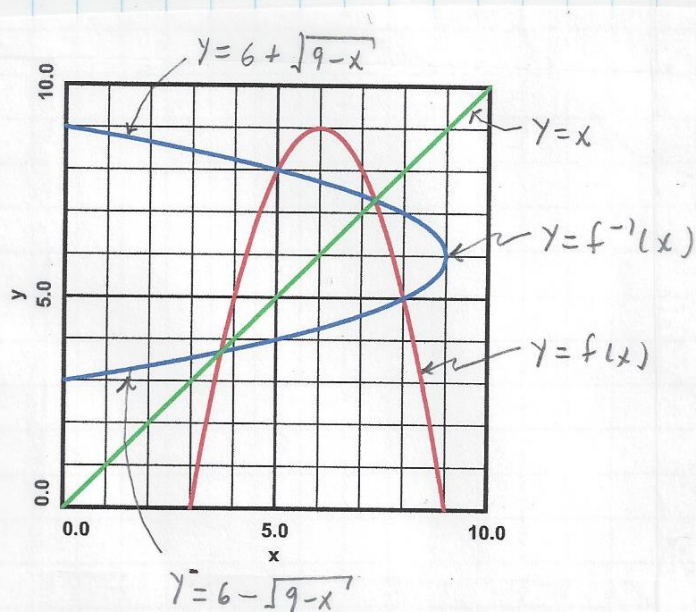
-36 +36

$$y = f(x) = -(x - 6)^2 + 9. \text{ Switch } x \text{ and } y \Rightarrow x = -(y - 6)^2 + 9,$$

$$\text{Solve for } y \dots -(y - 6)^2 = x - 9, \quad (y - 6)^2 = 9 - x, \quad y - 6 = \pm \sqrt{9 - x}$$

$$y = f^{-1}(x) = 6 \pm \sqrt{9 - x}$$

(b)



28.2. Parabolas

2 of 2

Example #2. For $f(x) = -x^2 + 12x - 27$ and $f^{-1}(x) = 6 \pm \sqrt{9-x}$ from Example #1, verify that $f(f^{-1}(x)) = x$.

SOLUTION:

$$(6 \pm \sqrt{9-x})^2 = 36 \pm 12\sqrt{9-x} + 9-x = 45-x \pm 12\sqrt{9-x}$$

$$f(f^{-1}(x)) = f(f^{-1}) = -(f^{-1})^2 + 12f^{-1} - 27 =$$

$$= -(6 \pm \sqrt{9-x})^2 + 12(6 \pm \sqrt{9-x}) - 27 =$$

$$= -(45-x \pm 12\sqrt{9-x}) + 12(6 \pm \sqrt{9-x}) - 27 =$$

$$= -45 + x \mp 12\sqrt{9-x} + 72 \pm 12\sqrt{9-x} - 27 = x - 45 + 72 - 27 = x$$