28.4. Definition of Logarithms

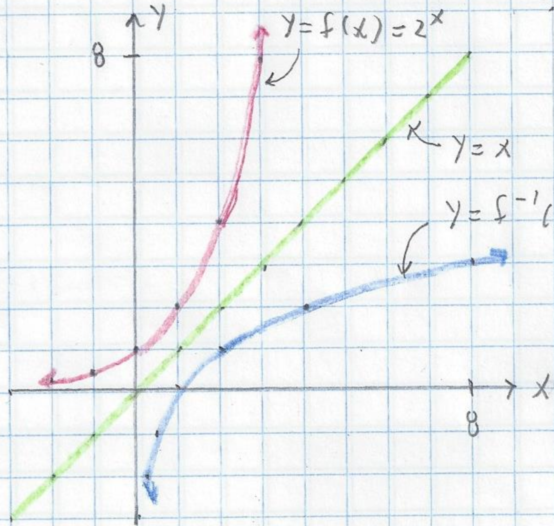
Consider the function $y = f(x) = 2^x$...

x	$y = f(x) = 2^x$
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8

$\rightarrow$ switch x and y to get the points on $y = f^{-1}(x)$ $\rightarrow$

x	$y = f^{-1}(x)$
0.25	-2
0.5	-1
1	0
2	1
4	2
8	3

Now, graph $y = f(x)$ and $y = f^{-1}(x)$...



$f(x) = 2^x$
exponential function of base 2

$f^{-1}(x) = \log_2 x$
logarithmic function of base 2
("log base 2")

$\boxed{\text{The inverse function of an exponential is a logarithm}}$

In general, for $N > 0$...

$f(x) = N^x$ (exponential of base N)

$f^{-1}(x) = \log_N x$ (log base N of x)

Note: The domain of $y = \log_N x$ is $0 < x < \infty$. So, for example, both $\log_N 0$ and $\log_N (-10)$ are undefined.

28.4. Definition of Logarithms

Inverse Function Properties

$f(f^{-1}(x)) = x \qquad N^{f^{-1}} = N^{\log_N x} = x$

$f^{-1}(f(x)) = x \qquad \log_N f = \log_N N^x = x$

$$\boxed{\begin{array}{c} \cancel{N}^{\cancel{\log_N} x} = x \\ \cancel{\log_N} \cancel{N}^{x} = x \end{array}}$$

Example #1. Convert the exponential equations to logarithmic form.

(a) $2^x = 16$ (b) $3^x = 2187$

SOLUTION:

(a) $\log_2 \cancel{2}^x = \log_2 16$, $x = \log_2 16$

(b) $\log_3 3^x = \log_3 2187$, $x = \log_3 2187$

Example #2. Convert the logarithmic equations to exponential form.

(a) $x = \log_8 512$ (b) $x = \log_5 625$

SOLUTION:

(a) $8^x = \cancel{8}^{\log_8 512}$, $8^x = 512$

(b) $5^x = \cancel{5}^{\log_5 625}$, $5^x = 625$

Example #3. Use the Inverse Function Properties to solve the following equations for x.

(a) $\log_2 x = 4$ (b) $\log_x 9 = 2$

SOLUTION:

(a) $\cancel{2}^{\log_2 x} = 2^4$, $x = 2^4 = 16$

(b) $\cancel{x}^{\log_x 9} = x^2$, $9 = x^2$, $x = 3$