28.6. Solving Exponential Equations

Example #1. Solve $2^{3x} \cdot 3^{4x} = 45$ for x.

SOLUTION:

$2^{3x} \cdot 3^{4x} = (2^3)^x \cdot (3^4)^x = 8^x \cdot 81^x = (8 \cdot 81)^x = 648^x = 45$,

$\log_{648} 648^x = \log_{648} 45$, $x = \log_{648} 45 = 0.5880$ ◄

Example #2. Solve $10^x + 10^{-x} = 3$ for x.

SOLUTION:

$10^x \cdot 10^x + 10^x \cdot 10^{-x} = 3 \cdot 10^x$ $\quad (10^x \cdot 10^{-x} = 10^{x-x} = 10^0 = 1)$

$(10^x)^2 + 1 = 3 \cdot 10^x$, $(10^x)^2 - 3(10^x) + 1 = 0$.

Let $z = 10^x$ (or $x = \log z$) $\Rightarrow$ $z^2 - 3z + 1 = 0$

$$z = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{5}}{2}$$

$x = \log\left(\frac{3 - \sqrt{5}}{2}\right) = -0.4180$ ◄

$x = \log\left(\frac{3 + \sqrt{5}}{2}\right) = 0.4180$ ◄

Example #3. Given $y = f(x)$, calculate $y = f^{-1}(x)$.

(a) $y = f(x) = e^{3x+8} - 4$ $\quad$ (b) $y = f(x) = \log(5x + 8) - 1$

SOLUTION:

(a) $x = e^{3y+8} - 4$, $e^{3y+8} = x + 4$, $\ln e^{3y+8} = \ln(x+4)$, $3y + 8 = \ln(x+4)$,

$3y = \ln(x+4) - 8$, $y = f^{-1}(x) = \frac{1}{3}\left[\ln(x+4) - 8\right]$ ◄

(b) $x = \log(5y + 8) - 1$, $\log(5y + 8) = x + 1$, $10^{\log(5y+8)} = 10^{x+1}$,

$5y + 8 = 10^{x+1}$, $5y = 10^{x+1} - 8$, $y = f^{-1}(x) = \frac{1}{5}\left[10^{x+1} - 8\right]$ ◄