

3A.1. Law of Exponents

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(1) $\underbrace{x \cdot x \cdot x}_{x^3} \cdot \underbrace{x \cdot x}_{x^2} = x^3 \cdot x^2 = x^5$ (add)

$$x^a \cdot x^b = x^{a+b}$$

(2) $\frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} = x^3 = \frac{x^5}{x^2}$ (subtract)

$$\frac{x^a}{x^b} = x^{a-b}$$

(3) $\frac{x^5}{x^5} = x^{5-5} = x^0 = 1$

$$x^0 = 1$$

(4) $\frac{1}{x^5} = \frac{x^0}{x^5} = x^{0-5} = x^{-5}$

$$x^{-a} = \frac{1}{x^a}$$

(5) $\frac{1}{x^{-5}} = \frac{x^0}{x^{-5}} = x^{0-(-5)} = x^5$

$$\frac{1}{x^{-a}} = x^a$$

(6) $\underbrace{(x \cdot x \cdot x)}_{x^3} \cdot \underbrace{(x \cdot x \cdot x)}_{x^3} = x^6 = (x^3)^2$ (multiply)

$$(x^a)^b = x^{a \cdot b}$$

(7) $(x^{\frac{1}{2}})^2 = x^{\frac{1}{2} \cdot 2} = x^1 = x$

$$x^{\frac{1}{a}} = \sqrt[a]{x}$$

$(\sqrt{x})^2 = x, x^{\frac{1}{2}} = \sqrt{x}$

(8) $(x^a)^{\frac{1}{b}} = x^{a \cdot \frac{1}{b}} = x^{\frac{a}{b}} = \sqrt[b]{x^a}$

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

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Example #1. Simplify each expression. Express your answers using only positive exponents

$$(a) \frac{2^{-3}x^4(y^2z^6)^3}{2^2x^3y^3z^{-2}}$$

$$(b) \frac{9x^{-8}(\sqrt[3]{y^2z})^3}{27x^2y^2z^6}$$

Solution:

$$(a) \frac{2^{-3}x^4(y^2z^6)^3}{2^2x^3y^3z^{-2}} = \frac{2^{-3}x^4y^6z^{18}}{2^2x^3y^3z^{-2}} = 2^{-3-2}x^{4-3}y^{6-3}z^{18-(-2)} = \\ = 2^{-5}x^1y^3z^{20} = \frac{1}{2^5}x^1y^3z^{20} = \frac{1}{32}x^1y^3z^{20} \leftarrow$$

$$(b) \frac{9x^{-8}(\sqrt[3]{y^2z})^3}{27x^2y^2z^6} = \frac{3^2x^{-8}(y^{\frac{2}{3}}z)^3}{3^3x^2y^2z^6} = \frac{3^2x^{-8}y^2z^3}{3^3x^2y^2z^6} = \\ = 3^{2-3}x^{-8-2}y^{2-2}z^{3-6} = 3^{-1}x^{-10}y^0z^{-3} = \frac{1}{3x^{10}z^3} \leftarrow$$

Example #2. Combine the radicals into a single radical.

$$(a) \frac{\sqrt[7]{x^4}}{\sqrt[8]{x^3}}$$

$$(b) \sqrt[6]{x} \cdot \sqrt[5]{x^2}$$

Solution:

$$(a) \frac{\sqrt[7]{x^4}}{\sqrt[8]{x^3}} = \frac{x^{4/7}}{x^{3/8}} = x^{\frac{4}{7}-\frac{3}{8}} = x^{\frac{11}{56}} = \sqrt[56]{x^{11}} \leftarrow$$

$$(b) \sqrt[6]{x} \cdot \sqrt[5]{x^2} = x^{\frac{1}{6}} \cdot x^{\frac{2}{5}} = x^{\frac{1}{6}+\frac{2}{5}} = x^{\frac{17}{30}} = \sqrt[30]{x^{17}} \leftarrow$$