3A.2. Transformation of Exponential Functions

Example #1. Consider $y = f(x) = 3^x$, $y = g(x) = 3^{x-2}$ and $y = h(x) = 3^{x+2}$.

(a) State the transformation that takes $y = f(x)$ to $y = g(x)$.
(b) State the transformation that takes $y = f(x)$ to $y = h(x)$.
(c) Write $y = g(x)$ as a vertical compression of $y = f(x)$.
(d) Write $y = h(x)$ as a vertical stretch of $y = f(x)$.
(e) On the same axes, graph $y = f(x)$, $y = g(x)$ and $y = h(x)$.

SOLUTION:

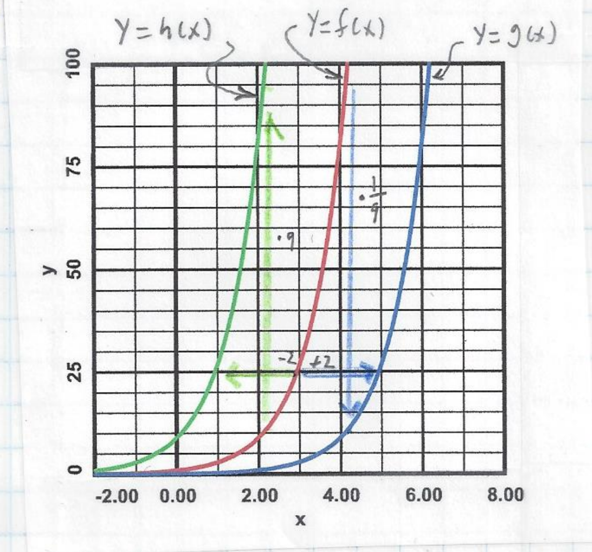
(a) Translate $y = f(x)$ by 2 right to get $y = g(x)$.

(b) Translate $y = f(x)$ by 2 left to get $y = h(x)$

(c) $y = g(x) = 3^{x-2} = 3^x \cdot 3^{-2} = \frac{1}{9} \cdot 3^x = \frac{1}{9} f(x)$

(d) $y = h(x) = 3^{x+2} = 3^x \cdot 3^2 = 9 \cdot 3^x = 9 f(x)$

(e)



Example #2. Let $y=f(x)=9^x$ be the parent function.

(a) Write $y=g(x)=81^x$ as a horizontal compression of the parent function.

(b) Write $y=h(x)=3^x$ as a horizontal stretch of the parent function.

SOLUTION:

(a) $y=g(x)=81^x=(9^2)^x=9^{2x}=f(2x)$

which is a horizontal compression, by a factor of $\frac{1}{2}$, of $y=f(x)$.

(b) $y=h(x)=3^x=(\sqrt{9})^x=(9^{1/2})^x=9^{\frac{x}{2}}=f\left(\frac{1}{2}x\right)$

which is a horizontal stretch, by a factor of 2, of $y=f(x)$.