3.4. Power Rule and Change of Base Formula

Power Rule

$x = N^p$, $\log_N x = \log_N N^p$, $p = \log_N x$

$x^q = (N^p)^q = N^{pq}$, $\log_N x^q = \log_N N^{pq} = pq$, $\log_N x^q = q\,p$

$$\log_N x^q = q \log_N x$$

Change of Base Formula

$x = N^p$, $p = \log_N x$

$\log_M x = \log_M N^p = p \log_M N = \log_N x \cdot \log_M N \Rightarrow$

$$\log_N x = \frac{\log_M x}{\log_M N}$$

In particular:

$M = 10 \longrightarrow \log_N x = \dfrac{\log x}{\log N}$

$M = e \longrightarrow \log_N x = \dfrac{\ln x}{\ln N}$

Example #1. Use the Power Rule to rewrite each logarithm.

(a) $\log_3 x^5$ (b) $\log_7 \sqrt[4]{x}$

SOLUTION:

(a) $\log_3 x^5 = 5 \log_3 x$ ◀ (b) $\log_7 \sqrt[4]{x} = \log_7 x^{1/4} = \frac{1}{4} \log_7 x$ ◀

Example #2. Write each expression as a single logarithm.

(a) $\dfrac{\log_7 82}{\log_7 4}$ (b) $\dfrac{\log_3 72}{\log_3 5}$

SOLUTION: Use the Change of Base Formula ---

(a) $\dfrac{\log_7 82}{\log_7 4} = \log_4 82$ ◄ (b) $\dfrac{\log_3 72}{\log_3 5} = \log_5 72$ ◄

Example #3. Calculate $\log_3 20$ by

(a) using common logarithms

(b) using natural logarithms

SOLUTION:

(a) $\log_3 20 = \dfrac{\log 20}{\log 3} = \dfrac{1.301030}{0.477121} = 2.7268$ ◄

(b) $\log_3 20 = \dfrac{\ln 20}{\ln 3} = \dfrac{2.995732}{1.098612} = 2.7268$ ◄