

3A.5. Product and Quotient Rules

1 of 2

Product Rule

$$x = N^p, \quad \log_N x = \log_N N^p = p, \quad p = \log_N x$$

$$y = N^q,$$

$$q = \log_N y$$

$$\log_N (xy) = \log_N (N^p N^q) = \log_N N^{p+q} = p+q = \log_N x + \log_N y$$

$$\boxed{\log_N (xy) = \log_N x + \log_N y}$$

Quotient Rule

$$x = N^p, \quad p = \log_N x$$

$$y = N^q, \quad q = \log_N y$$

$$\log_N \left(\frac{x}{y} \right) = \log_N \left(\frac{N^p}{N^q} \right) = \log_N N^{p-q} = p-q = \log_N x - \log_N y$$

$$\boxed{\log_N \left(\frac{x}{y} \right) = \log_N x - \log_N y}$$

Example #1. Express $4\log x + 3\log y - 7\log z$ as a single logarithm.

Solution:

$$4\log x + 3\log y - 7\log z = \log x^4 + \log y^3 - \log z^7 = \log (x^4 y^3) - \log z^7 =$$

$$= \log \left(\frac{x^4 y^3}{z^7} \right)$$

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20#2

Example #2, Expand $\ln \sqrt[7]{\frac{xy^4}{z^3}}$.

SOLUTION:

$$\ln \sqrt[7]{\frac{xy^4}{z^3}} = \ln \left(\frac{xy^4}{z^3} \right)^{1/7} = \frac{1}{7} \ln \left(\frac{xy^4}{z^3} \right) = \frac{1}{7} \left[\ln(xy^4) - \ln z^3 \right] =$$

$$= \frac{1}{7} \left[\ln x + \ln y^4 - \ln z^3 \right] = \frac{1}{7} \left[\ln x + 4 \ln y - 3 \ln z \right] = \frac{1}{7} \ln x + \frac{4}{7} \ln y - \frac{3}{7} \ln z$$