3A.6. Transformation of Logarithmic Functions

Example #1. Let $y=f(x)=\log_2 x$, $y=g(x)=\log_2 x+4$, $y=h(x)=\log_2 x-4$.

(a) Describe the translation that takes $y=f(x)$ to $y=g(x)$.

(b) Describe the translation that takes $y=f(x)$ to $y=h(x)$.

(c) Write $y=g(x)$ as a horizontal compression of $y=f(x)$.

(d) Write $y=h(x)$ as a horizontal stretch of $y=f(x)$.

(e) Graph $y=f(x)$, $y=g(x)$ and $y=h(x)$ on the same set of axes.

SOLUTION:

(a) $y=g(x)=f(x)+4$ is a translation of $y=f(x)$ by 4 up ←

(b) $y=h(x)=f(x)-4$ is a translation of $y=f(x)$ by 4 down ←

(c) $4=\log_2 w$, $2^4=2^{\log_2 w}$, $w=2^4=16$, $4=\log_2 16$

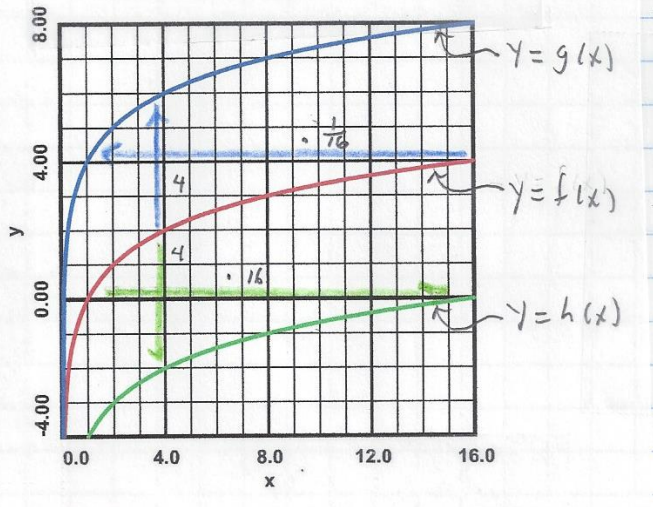
$y=g(x)=\log_2 x+4=\log_2 x+\log_2 16=\log_2(16x)=f(16x)$ ←

which is a horizontal compression, by a factor of $\frac{1}{16}$, of $y=f(x)$.

(d) $y=h(x)=\log_2 x-4=\log_2 x-\log_2 16=\log_2\left(\frac{x}{16}\right)=f\left(\frac{1}{16}x\right)$ ←

which is a horizontal stretch, by a factor of 16, of $y=f(x)$.

(e)



Example 2. Describe the transformation that takes $y=f(x)=\log x$ to $y=g(x)=\log(x-5)^4+7$.

SOLUTION:

$y=g(x)=4\log(x-5)+7=4f(x-5)+7$.

Vertically stretch $y=f(x)$ by a factor of 4, and then translate 5 right and 7 up ◄

Example #3. Let $y=f(x)=\log_2 x$.

(a) Write $y=g(x)=\log_2 x^3$ as a vertical stretch of $y=f(x)$.

(b) Write $y=h(x)=\log_2 \sqrt[3]{x}$ as a vertical compression of $y=f(x)$.

SOLUTION:

(a) $y=g(x)=\log_2 x^3=3\log_2 x=3f(x)$ ◄

(b) $y=h(x)=\log_2 \sqrt[3]{x}=\log_2 x^{1/3}=\frac{1}{3}\log_2 x=\frac{1}{3}f(x)$ ◄

Example #4. For $y=f(x)=\log_2(x-4)$,

(a) Graph $y=f(x)$ and its vertical asymptote (V.A.)

(b) Calculate $y=f^{-1}(x)$.

(c) Graph $y=f^{-1}(x)$ and its horizontal asymptote (H.A.)

(d) State the domain and range of both $y=f(x)$ and $y=f^{-1}(x)$.

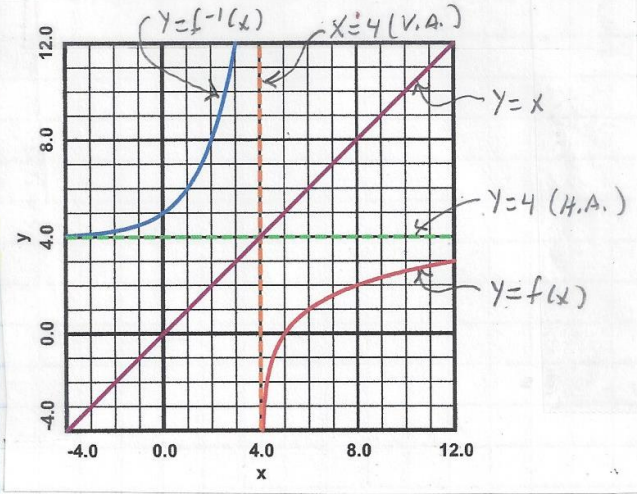
SOLUTION:

(b) $y=\log_2(x-4)$, $x=\log_2(y-4)$,

$2^x=2^{\log_2(y-4)}$, $y-4=2^x$,

$y=f^{-1}(x)=2^x+4$ ◄

(a) (c)



(d) $y=f(x)$:

domain: $4<x<\infty$ ◄

range: $-\infty<y<\infty$ ◄

$y=f^{-1}(x)$

domain: $-\infty<x<\infty$ ◄

range: $4<y<\infty$ ◄