

3A.7. Solving Logarithmic Equations

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Example #1. Solve $\log_3(x-1) + \log_3(x-7) = 2$ for x .

SOLUTION:

$$\log_3[(x-1)(x-7)] = 2, \quad \log_3[x^2 - 8x + 7] = 2$$

$$3^{\log_3[x^2 - 8x + 7]} = 3^2 = 9, \quad x^2 - 8x + 7 = 9, \quad x^2 - 8x - 2 = 0$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-2)}}{2(1)} = \frac{8 \pm \sqrt{72}}{2} = \frac{8 \pm \sqrt{36} \sqrt{2}}{2} = \frac{8 \pm 6\sqrt{2}}{2} = 4 \pm 3\sqrt{2}$$

$$x = 4 - 3\sqrt{2} = -0.2426 \quad \times \text{ gives log of negative numbers}$$

$$x = 4 + 3\sqrt{2} = 8.2426 \quad \checkmark$$

Example #2. Solve $\log_2(x^2 + 3x - 10) = 8 + \log_2(x-2)$ for x .

SOLUTION:

$$\log_2[(x+5)(x-2)] = 8 + \log_2(x-2), \quad \log_2(x+5) + \log_2(x-2) = 8 + \log_2(x-2)$$

$$2^{\log_2(x+5)} = 2^8 = 256, \quad x+5 = 256, \quad x = 251 \quad \checkmark$$

Example #3. Solve $\log_5(x+7) - \log_5(x-3) = 3$ for x .

SOLUTION:

$$\log_5\left(\frac{x+7}{x-3}\right) = 3, \quad 5^{\log_5\left(\frac{x+7}{x-3}\right)} = 5^3 = 125,$$

$$\frac{x+7}{x-3} = 125, \quad x+7 = 125(x-3) = 125x - 375, \quad 124x = 382,$$

$$x = 3\frac{5}{62} \quad \checkmark$$