Homework #3A

Lesson 3A.1:

1) $\dfrac{81x^7\left(y^2\cdot\sqrt[3]{z^5}\right)^3}{3x^{-2}y^3z^5} = \dfrac{3^4x^7\left(y^2z^{5/3}\right)^3}{3x^{-2}y^3z^5} = \dfrac{3^4x^7y^6z^5}{3x^{-2}y^3z^5} = 3^{4-1}x^{7--2}y^{6-3}z^{5-5} =$

$= 3^3x^9y^3z^0 = 27x^9y^3$ ◄

2) $\dfrac{\sqrt[8]{x}}{\sqrt[4]{x^3}} = \dfrac{x^{1/8}}{x^{3/4}} = x^{\frac{1}{8}-\frac{3}{4}} = x^{-5/8} = \dfrac{1}{x^{5/8}} = \dfrac{1}{\sqrt[8]{x^5}}$ ◄

Lesson 3A.2:

3) a) Translate $y=f(x)$ 4 right to get $y=g(x)$. ◄

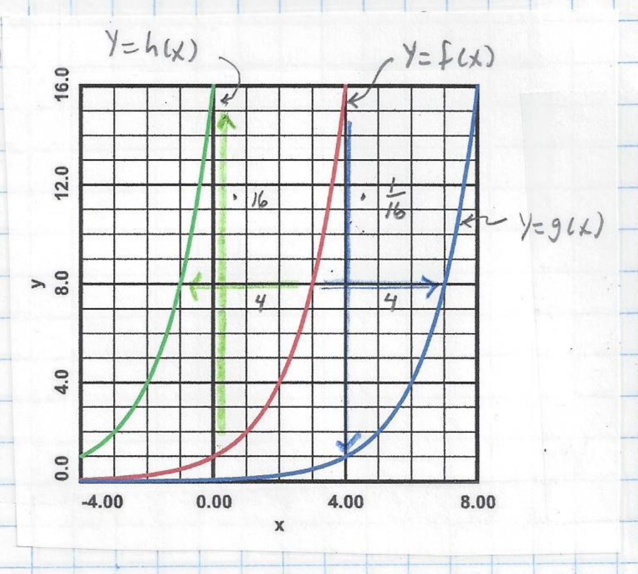
b) Translate $y=f(x)$ 4 left to get $y=h(x)$. ◄

c) $y=g(x)=2^{x-4}=2^x\cdot2^{-4}=\frac{1}{16}\cdot2^x=\frac{1}{16}f(x)$. ◄

Vertically compress $y=f(x)$, by a factor of $\frac{1}{16}$, to obtain $y=g(x)$. ◄

d) $y=h(x)=2^{x+4}=2^x\cdot2^4=16\cdot2^x=16f(x)$. ◄

Vertically stretch $y=f(x)$, by a factor of 16, to obtain $y=f(x)$. ◄

e) 

4) $y=f(x)=25^x$

a) $y=g(x)=5^x=\left(\sqrt{25}\right)^x=$

$=\left(25^{\frac{1}{2}}\right)^x=25^{\frac{x}{2}}=$

$=f\left(\frac{1}{2}x\right)$ ◄

Horizontally stretch $y=f(x)$, by a factor of 2, to obtain $y=g(x)$. ◄

b) $y=h(x)=625^x=$

$=(25^2)^x=25^{2x}=f(2x)$. ◄

Horizontally compress $y=f(x)$, by a factor of $\frac{1}{2}$, to obtain $y=h(x)$. ◄

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Lesson 3A.3:

5) a) $y=y(x)$ is exponential ◄ One sees that $y=3^x$ fits the table.

b) The y-values increase 5 each time $\rightarrow$ $y=y(x)$ is linear ◄
One sees that $y=5x+8$ fits the table.

c) This is the inverse function of part (a) $\rightarrow$ $y=y(x)$ is logarithmic ◄
One sees that $y=\log_3 x$ fits the table.

d) The graph is a parabola with vertex $(5,-2)$ $\rightarrow$ $y=y(x)$ is quadratic ◄
One sees that $y=(x-5)^2-2=x^2-10x+25-2=x^2-10x+23$ fits the table.

Lesson 3A.4:

6) $\log_{21}\sqrt[13]{x^8}=\log_{21}x^{\frac{8}{13}}=\frac{8}{13}\log_{21}x$ ◄

7) a) $\log_9 53=\dfrac{\log 53}{\log 9}=\dfrac{1.724276}{0.954243}=1.8070$ ◄

b) $\log_9 53=\dfrac{\ln 53}{\ln 9}=\dfrac{3.970292}{2.197225}=1.8070$ ◄

Lesson 3A.5:

8) $3\log_4 x-7\log_4 y-9\log_4 z=\log_4 x^3-\log_4 y^7-\log_4 z^9=$
$=\log_4 x^3-\left(\log_4 y^7+\log_4 z^9\right)=\log_4 x^3-\log_4\left(y^7z^9\right)=\log_4\left(\dfrac{x^3}{y^7z^9}\right)$ ◄

9) $\log_6\sqrt[7]{\dfrac{x^5}{y^3z^2}}=\log_6\left(\dfrac{x^5}{y^3z^2}\right)^{1/7}=\frac{1}{7}\log_6\left(\dfrac{x^5}{y^3z^2}\right)=\frac{1}{7}\left\{\log_6 x^5-\log_6\left(y^3z^2\right)\right\}=$

$=\frac{1}{7}\left\{\log_6 x^5-\left(\log_6 y^3+\log_6 z^2\right)\right\}=\frac{1}{7}\left\{\log_6 x^5-\log_6 y^3-\log_6 z^2\right\}=$

$=\frac{1}{7}\left\{5\log_6 x-3\log_6 y-2\log_6 z\right\}=\frac{5}{7}\log_6 x-\frac{3}{7}\log_6 y-\frac{2}{7}\log_6 z$ ◄

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Lesson 3A.6:

10) a) Translate $y=f(x)$ 3 up to obtain $y=g(x)$.

b) Translate $y=f(x)$ 3 down to obtain $y=h(x)$.

c) $3=\log_3 w$, $w=3^3=27$, $3=\log_3 27$

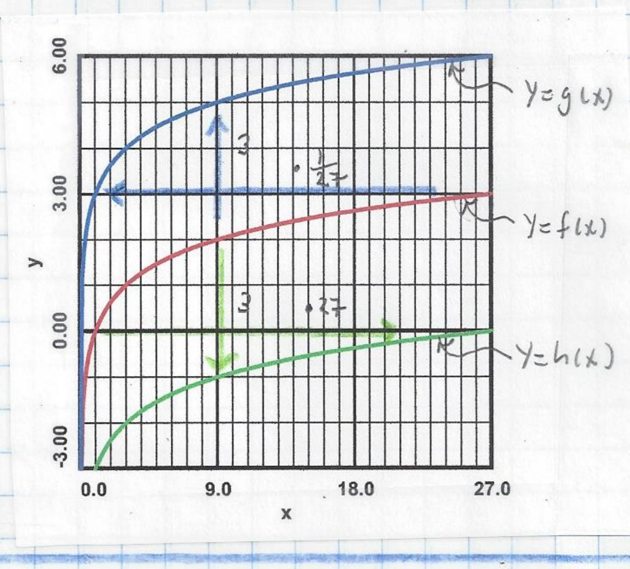
$y=g(x)=\log_3 x+3=\log_3 x+\log_3 27=\log_3(27x)=f(27x)$

Horizontally compress $y=f(x)$, by a factor of $\frac{1}{27}$, to obtain $y=g(x)$.

d) $y=h(x)=\log_3 x-3=\log_3 x-\log_3 27=\log_3\left(\frac{x}{27}\right)=f\left(\frac{1}{27}x\right)$

Horizontally stretch $y=f(x)$, by a factor of 27, to obtain $y=h(x)$

e)



11) $y=f(x)=\log_6 x$

a) $y=g(x)=\log_6 x^4=$

$=4\log_6 x=4f(x)$

vertically stretch $y=f(x)$, by a factor of 4, to obtain $y=g(x)$.

b) $y=h(x)=\log_6 \sqrt[4]{x}=$

$=\log_6 x^{\frac{1}{4}}=\frac{1}{4}\log_6 x=$

$=\frac{1}{4}f(x)$

vertically compress $y=f(x)$, by a factor of $\frac{1}{4}$, to obtain $y=h(x)$.

12) c) $x=2\log_2(y-4)+3$, $2\log_2(y-4)=x-3$, $\log_2(y-4)=\frac{1}{2}(x-3)$,

$y-4=2^{\frac{1}{2}(x-3)}$, $y=f^{-1}(x)=2^{\frac{1}{2}(x-3)}+4$

e) $y=f(x)$: domain: $4<x<\infty$ range: $-\infty<y<\infty$

$y=f^{-1}(x)$: domain: $-\infty<x<\infty$ range: $4<y<\infty$

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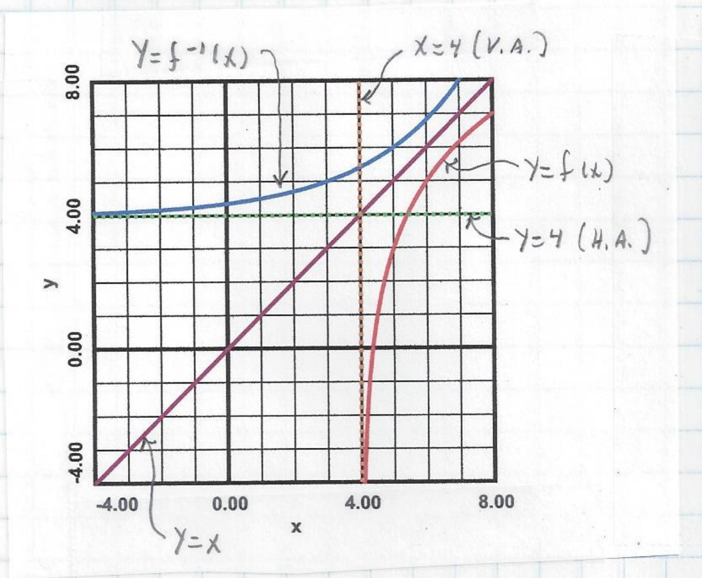
12) a), b), d)



Lesson 3A.7

13)

a) $\log_4(x+9)+\log_4(x-3)=3$

$4^{\log_4[(x+9)(x-3)]}=4^3$

$(x+9)(x-3)=4^3=64$

$x^2+6x-27=64,$

$x^2+6x-91=0$

$x=\dfrac{-6\pm\sqrt{6^2-4(1)(-91)}}{2(1)}=$

$=\dfrac{-6\pm\sqrt{400}}{2}=\dfrac{-6\pm20}{2}=$

$=-3\pm10$

$x=-13$ ✗ $\quad x=7$ ✓

b) $\log_3(x+8)-\log_3(x-2)=4,\quad 3^{\log_3\left(\frac{x+8}{x-2}\right)}=3^4,\quad \dfrac{x+8}{x-2}=3^4=81,$

$x+8=81(x-2)=81x-162,\quad 80x=170,\quad x=2\frac{1}{8}$ ✓

Lesson 3A.8:

14) $2000\to t=0,\ 2020\to t=20,\ p=p_0e^{kt},\ p=144{,}174e^{kt},$

$180{,}355=144{,}174e^{k(20)},\quad e^{20k}=\dfrac{180{,}355}{144{,}174},\quad 20k=\ln\left(\dfrac{180{,}355}{144{,}174}\right)$

$k=\dfrac{1}{20}\ln\left(\dfrac{180{,}355}{144{,}174}\right)=0.011195\ldots/\text{yr},\quad 215{,}000=144{,}174e^{kt},$

$e^{kt}=\dfrac{215{,}000}{144{,}174},\quad kt=\ln\left(\dfrac{215{,}000}{144{,}174}\right),\quad t=\dfrac{1}{k}\ln\left(\dfrac{215{,}000}{144{,}174}\right)=35.7\to2036$ ◄

15) $k=\dfrac{\ln2}{T}=\dfrac{\ln2}{8}=0.08664\ldots/\text{day},\quad c=c_0e^{-kt},$ 95% decay → 5% left

$0.05c_0=c_0e^{-kt},\quad 0.05=e^{-kt},\quad -kt=\ln(0.05),\quad t=-\dfrac{1}{k}\ln(0.05)=34.58$ days ◄