3B.1. Factoring Quadratics

I. $a = 1$, $f(x) = x^2 + bx + c$

$$f(x) = (x+t)(x+u) = x^2 + ux + tx + tu = x^2 + \overbrace{(t+u)}^{b}x + \overbrace{(tu)}^{c}$$

So, find two numbers t and u such that $b = t+u$, $c = tu$.

Example #1. Factor

(a) $f(x) = x^2 + 11x + 28$ (b) $f(x) = x^2 - 3x - 10$

SOLUTION:

(a) $t+u = 11$, $tu = 28 \rightarrow t = 7$, $u = 4 \rightarrow f(x) = (x+7)(x+4)$

(b) $t+u = -3$, $tu = -10 \rightarrow t = -5$, $u = 2 \rightarrow f(x) = (x-5)(x+2)$

Example #2. Solve the quadratic equations for x by factoring.

(a) $x^2 + 11x + 28 = 0$ (b) $x^2 - 3x - 10 = 0$

SOLUTION:

(a) From Example #1(a), $x^2 + 11x + 28 = (x+7)(x+4) = 0$

$x+7 = 0$, $x = -7$ $x+4 = 0$, $x = -4$

(b) From Example #1(b), $x^2 - 3x - 10 = (x-5)(x+2) = 0$

$x-5 = 0$, $x = 5$ $x+2 = 0$, $x = -2$

II. $a \neq 1$, $f(x) = ax^2 + bx + c$

$$f(x) = (px+q)(rx+s) = prx^2 + psx + qrx + qs = \overbrace{(pr)}^{a}x^2 + \overbrace{(ps+qr)}^{b}x + \overbrace{(qs)}^{c}$$

$$b = \underbrace{ps}_{t} + \underbrace{qr}_{u}, \quad ac = (pr)(qs) = \underbrace{(ps)}_{t}\underbrace{(qr)}_{u}$$

So, find two numbers t and u such that $ac = tu$, $b = t+u$.

Example #3. Factor

(a) $f(x) = 6x^2 - x - 40$ (b) $f(x) = 28x^2 + 59x - 9$

SOLUTION:

(a) $ac = (6)(-40) = -240 = -2^4 \cdot 3 \cdot 5 = tu$, $b = -1 = t + u$, $t = -16$, $u = 15$

- Break up the middle term according to t and u...

$f(x) = 6x^2 \underline{-x} - 40 = 6x^2 \underline{-16x + 15x} - 40$

- Next, factor out what you can from the first two terms and the second two terms...

$f(x) = \underline{6x^2 - 16x} + \underline{15x - 40} = \underline{2x(3x-8)} + \underline{5(3x-8)} = (2x+5)(3x-8)$ ◄

(b) $ac = (28)(-9) = -252 = -2^2 \cdot 3^2 \cdot 7 = tu$, $b = 59 = t + u$, $t = 63$, $u = -4$

$f(x) = 28x^2 + 59x - 9 = \underline{28x^2 + 63x} \underline{- 4x - 9} = \underline{7x(4x+9)} \underline{-1(4x+9)} =$

$= (7x-1)(4x+9)$ ◄

Example #4. Solve the quadratic equations for x by factoring.

(a) $6x^2 - x - 40 = 0$ (b) $28x^2 + 59x - 9 = 0$

SOLUTION:

(a) From Example #3(a), $6x^2 - x - 40 = (2x+5)(3x-8) = 0 \rightarrow$

$2x + 5 = 0$, $2x = -5$, $x = -\frac{5}{2}$ ◄ $3x - 8 = 0$, $3x = 8$, $x = \frac{8}{3}$ ◄

(b) From Example #3(b), $28x^2 + 59x - 9 = (7x-1)(4x+9) = 0 \rightarrow$

$7x - 1 = 0$, $7x = 1$, $x = \frac{1}{7}$ ◄ $4x + 9 = 0$, $4x = -9$, $x = -\frac{9}{4}$ ◄