3B.2. Complex Numbers

$i \equiv$ the imaginary unit

$i = \sqrt{-1}$, $i^2 = -1$

Example #1. Evaluate

(a) $\sqrt{-121}$ (b) $(2i)^2$ (c) $\sqrt{-9} \cdot \sqrt{-4}$

SOLUTION:

(a) $\sqrt{-121} = \sqrt{121}\,\sqrt{-1} = 11\,i$

(b) $(2i)^2 = 2^2 i^2 = (4)(-1) = -4$

(c) $\sqrt{-9} \cdot \sqrt{-4} = 3i \cdot 2i = 6i^2 = -6$

Complex Numbers

$z = x + yi$, $z \equiv$ complex number
$x \equiv$ real part of z
$y \equiv$ imaginary part of z

Add, Subtract, Multiply

Example #2. For $z_1 = 2 + 3i$ and $z_2 = 4 - 5i$, calculate

(a) $z_1 + z_2$ (b) $z_1 - z_2$ (c) $z_1 z_2$

SOLUTION:

(a) $z_1 + z_2 = (2+3i) + (4-5i) = 2 + 4 + 3i - 5i = 6 - 2i$

(b) $z_1 - z_2 = (2+3i) - (4-5i) = 2 - 4 + 3i + 5i = -2 + 8i$

(c) $z_1 z_2 = (2+3i)(4-5i) = 8 - 10i + 12i - 15i^2 = 8 + 15 - 10i + 12i = 23 + 2i$

3B.2. Complex Numbers

Conjugates: switch the sign of i...

Example #3. Find the conjugate of

(a) $4i$ (b) $2+3i$ (c) $4-5i$

SOLUTION:

(a) $-4i$ (b) $2-3i$ (c) $4+5i$

Example #4. Calculate $(4+7i)(4-7i)$.

SOLUTION:

$$(4+7i)(4-7i) = 16 - \cancel{28i} + \cancel{28i} - 49i^2 = 16+49 = 65$$

Fact: the product of conjugates is real.

Divide

Example #5. Calculate $\frac{2+3i}{4-5i}$.

SOLUTION:

$$\frac{(2+3i)}{(4-5i)} \cdot \frac{(4+5i)}{(4+5i)} = \frac{8+10i+12i+15i^2}{16+\cancel{20i}-\cancel{20i}-25i^2} = \frac{8+22i-15}{16+25} = \frac{-7+22i}{41} =$$

$$= -\frac{7}{41} + \frac{22}{41} i$$