3B.3. Quadratic Root Theorem

Roots

The solutions of $f(x) = ax^2 + bx + c = 0$ are called roots of $f(x)$.

Example #1: Find the roots and x- and y-intercepts of each quadratic function.

(a) $f(x) = x^2 - 4x + 13$ (b) $f(x) = x^2 - 6x + 2$

SOLUTION:

(a) $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} = \dfrac{4 \pm \sqrt{-36}}{2} = \dfrac{4 \pm 6i}{2} = 2 \pm 3i$

roots: $x = 2+3i$, $x = 2-3i$ ◂ x-ints: none ◂ y-int.: $y = 13$ ◂

(b) $x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)} = \dfrac{6 \pm \sqrt{28}}{2} = \dfrac{6 \pm \sqrt{4}\sqrt{7}}{2} = \dfrac{6 \pm 2\sqrt{7}}{2} = 3 \pm \sqrt{7}$

roots: $x = 3+\sqrt{7}$, $x = 3-\sqrt{7}$ ◂ x-ints: same as the roots ◂ y-int.: $y = 2$

Quadratic Root Theorem

The followin facts follow directly from the Quadratic Formula...

For p and q real, and $f(x)$ being a quadratic function:

(1) If $p + qi$ is a root of $f(x)$, then so is $p - qi$

(2) If $p + \sqrt{q}$ is a root of $f(x)$, then so is $p - \sqrt{q}$

Example #2. Construct a quadratic function $y=f(x)$ which has

(a) $x=4-5i$ (b) $x=7+\sqrt{5}$

as one of its roots. Also, verify that the roots obey $f(x)=0$.

SOLUTION:

(a) $x=4+5i$ is the other root

$f(x)=[x-(4-5i)][x-(4+5i)]=x^2-(4+5i)x-(4-5i)x+(4-5i)(4+5i)=$

$=x^2-4x-\cancel{5ix}-4x+\cancel{5ix}+16+\cancel{20i}-\cancel{20i}-25i^2=$

$=x^2-8x+16+25=x^2-8x+41$

$x^2=(4\pm 5i)^2=16\pm 40i+25i^2=-9\pm 40i$

$x^2-8x+41=(-9\pm 40i)-8(4\pm 5i)+41=-9\cancel{\pm 40i}-32\cancel{\mp 40i}+41=$

$=-9-32+41=0$ ✓

(b) $x=7-\sqrt{5}$ is the other root

$f(x)=[x-(7+\sqrt{5})][x-(7-\sqrt{5})]=x^2-(7-\sqrt{5})x-(7+\sqrt{5})x+(7+\sqrt{5})(7-\sqrt{5})=$

$=x^2-7x+\cancel{\sqrt{5}x}-7x-\cancel{\sqrt{5}x}+49-\cancel{7\sqrt{5}}+\cancel{7\sqrt{5}}-5=x^2-14x+44$

$x^2=(7\pm\sqrt{5})^2=49\pm 14\sqrt{5}+5=54\pm 14\sqrt{5}$

$x^2-14x+44=(54\pm 14\sqrt{5})-14(7\pm\sqrt{5})+44=54\cancel{\pm 14\sqrt{5}}-98\cancel{\mp 14\sqrt{5}}+44$

$=54-98+44=0$ ✓