3B.5. Solving Cubic Equations

Example #1. For $f(x) = 20x^3 - 43x^2 - 50x + 112$

(a) Graph $Y = f(x)$ with the window: $-3 \le x < 3$, $-10 \le y \le 130$, to find the integer root of $Y = f(x)$.

(b) Use synthetic division to help factor $y = f(x)$ into three linear factors.

(c) State three roots of $y = f(x)$.

SOLUTION:

(a) $x = 2$ is a root $\rightarrow$ $x - 2$ is a factor

(b)

2	20	−43	−50	112
		40	−6	−112
	20	−3	−56	0

$\longrightarrow f(x) = (x-2)(20x^2 - 3x - 56)$

$ac = (20)(-56) = -1120 = -2^5 \cdot 5 \cdot 7 = tu$, $b = -3 = t + u$, $t = -35$, $u = 32 \rightarrow$

$20x^2 - 3x - 56 = 20x^2 - 35x + 32x - 56 = 5x(4x-7) + 8(4x-7) = (5x+8)(4x-7)$,

$f(x) = (x-2)(5x+8)(4x-7)$ ◀

(c) $f(x) = 0 \Rightarrow$

$x - 2 = 0$, $x = 2$ ◀ $5x + 8 = 0$, $x = -\frac{8}{5}$ ◀, $4x - 7 = 0$, $x = \frac{7}{4}$ ◀

Example #2. Write $f(x) = (x-1)(2x+3)(4x-5)$ in standard form.

SOLUTION:

$(2x+3)(4x-5) = 8x^2 - 10x + 12x - 15 = 8x^2 + 2x - 15$

$f(x) = (x-1)(2x+3)(4x-5) = (x-1)(8x^2 + 2x - 15) =$

$= 8x^3 + 2x^2 - 15x$
$\quad -8x^2 - 2x + 15 = 8x^3 - 6x^2 - 17x + 15$ ◀

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Example #3. For

(i) $f(x) = x^3 - 7x^2 + 25x - 39$ (ii) $f(x) = x^3 - 11x^2 + 32x - 10$

(a) Graph $y = f(x)$ with the window: $0 \le x \le 7$, $-40 \le y \le 40$, to find the integer root of $y = f(x)$.

(b) Use synthetic division and the Quadratic Formula to find the other two roots of $y = f(x)$.

(c) State the x- and y-intercepts of $y = f(x)$.

SOLUTION:

(i)

(a) $x = 3$ is a root $\rightarrow$ $x - 3$ is a factor

(b)
$$\begin{array}{r|rrr|r} 3 & 1 & -7 & 25 & -39 \\ & & 3 & -12 & 39 \\ \hline & 1 & -4 & 13 & 0 \end{array} \rightarrow f(x) = (x-3)(x^2 - 4x + 13)$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

The three roots are $x = 3$, $x = 2 + 3i$, $x = 2 - 3i$.

(c) x-ints: $x = 3$ y-int.: $y = -39$

(ii)

(a) $x = 5$ is a root $\rightarrow$ $x - 5$ is a factor

(b)
$$\begin{array}{r|rrr|r} 5 & 1 & -11 & 32 & -10 \\ & & 5 & -30 & 10 \\ \hline & 1 & -6 & 2 & 0 \end{array} \rightarrow f(x) = (x-5)(x^2 - 6x + 2)$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)} = \frac{6 \pm \sqrt{28}}{2} = \frac{6 \pm \sqrt{4}\sqrt{7}}{2} =$$

$$= \frac{6 \pm 2\sqrt{7}}{2} = 3 \pm \sqrt{7}.$$ The three roots are $x = 5$, $x = 3 + \sqrt{7}$, $x = 3 - \sqrt{7}$.

(c) x-ints: same as the three roots; y-int: $y = -10$