3B.8. Rational Functions

$$\boxed{R(x) = \frac{P(x)}{Q(x)}}$$

$R(x) \equiv$ rational function
$P(x) \equiv$ polynomial
$Q(x) \equiv$ polynomial

Horizontal Asymptote (H.A.)

Look at large x, or $x \to \pm\infty$ ("x approaches $\pm\infty$")

Example #1. Find the H.A. of

(a) $R(x) = \dfrac{x+8}{x^2+3x+2}$ (b) $R(x) = \dfrac{5x^2+7x+2}{x^2-5x+1}$

SOLUTION:

neglect for large x

(a) $R(x) = \dfrac{x+8}{x^2+3x+2}$

As $x \to \pm\infty$,

$R(x) \to \dfrac{x}{x^2} = \dfrac{1}{x} \to 0$

$y=0$ is H.A.

(b) $R(x) = \dfrac{5x^2+7x+2}{x^2-5x+1}$

As $x \to \pm\infty$,

$R(x) \to \dfrac{5x^2}{x^2} = 5$

$y=5$ is H.A.

Vertical Asymptotes (V.A.s)

If $P(x)$ and $Q(x)$ have no common factors, then $R(x) \to \pm\infty$ near the x-values where $Q(x)=0$, i.e., you are trying to divide by 0. These x-values are the V.A.s of $y=R(x)$.

Example #2. For $R(x) = \dfrac{x^2 - x - 6}{x^2 - 4x - 5}$, without using your calculator, find:

(a) The H.A. of $y = R(x)$.
(b) The V.A.s of $y = R(x)$
(c) The x-intercepts of $y = R(x)$
(d) The y-intercept of $y = R(x)$

SOLUTION: $R(x) = \dfrac{(x+2)(x-3)}{(x+1)(x-5)} = \dfrac{x^2 - x - 6}{x^2 - 4x - 5}$

(a) $R(x) = \dfrac{x^2 - x - 6}{x^2 - 4x - 5}$ neglect As $x \to \pm\infty$, $R(x) \to \dfrac{x^2}{x^2} = 1$, $y = 1$ is H.A.

(b) V.A.s are where the denominator is zero $\Rightarrow$ $x = -1$ & $x = 5$ are V.A.s

(c) x-ints. are where the numerator is zero $\Rightarrow$ x-int.: $x = -2$, $x = 3$

(d) y-int: $y = R(0) = \dfrac{-6}{-5} = \dfrac{6}{5} = 1.2$

Example #3. With your calculator, graph $y = R(x)$ from Example #2. Include the H.A., the V.A.s and x- and y-intercepts on the graph.

SOLUTION: