3C.2. Transformation of Square Root Functions

Example #1. Let $y = f(x) = 4 - \sqrt{4-x}$.

(a) Construct $y = g(x)$, which is a vertical compression, by a factor of $\frac{1}{2}$, of $y = f(x)$.

(b) Graph $y = f(x)$ and $y = g(x)$.

(c) Construct $y = h(x)$, which is a translation, by $(5, 3)$, of $y = g(x)$.

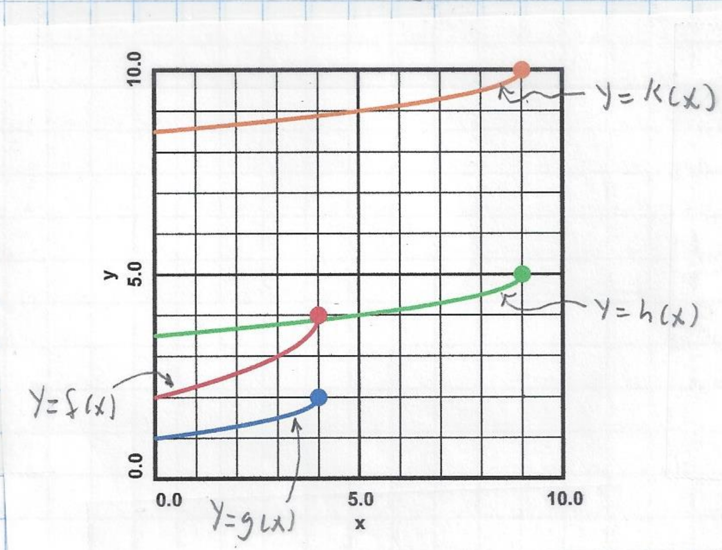
(d) Graph $y = h(x)$.

Solution:

(a) $y = g(x) = \frac{1}{2} f(x) = 2 - \frac{1}{2}\sqrt{4-x}$ ◄

(c) $y = h(x) = g(x-5) + 3 = 2 - \frac{1}{2}\sqrt{4-(x-5)} + 3 = 5 - \frac{1}{2}\sqrt{9-x}$ ◄

(b)
(d)



Example #2. The transformation of $y = f(x)$ to $y = h(x)$ in Example #1 is equivalent to:

(a) Construct a function $y = k(x)$, which is a translation, by $(5, 6)$, of $y = f(x)$.

(b) Graph $y = k(x)$ on the grid in Example #1.

(c) Construct $y = h(x)$ as a vertical compression, by a factor of $\frac{1}{2}$, of $y = k(x)$.

SOLUTION:

(a) $y = k(x) = f(x-5) + 6 = 4 - \sqrt{4-(x-5)} + 6 = 10 - \sqrt{9-x}$

(c) $y = h(x) = \frac{1}{2}k(x) = 5 - \frac{1}{2}\sqrt{9-x}$ (same as in Example #1).

Example #3. Let $y = f(x) = 7 - \sqrt{4x-11}$.

(a) Graph $y = f(x)$.

(b) State the domain and range of $y = f(x)$.

(c) Calculate $y = f^{-1}(x)$.

(d) On the same axes used in part (a), graph both $y = f^{-1}(x)$ and $y = x$.

SOLUTION:

(b) $4x - 11 = 0$, $4x = 11$, $x = \frac{11}{4} = 2.75$

x	$y = f(x)$
2	undef.
2.75	7 ← vertex
3	6

(2.75, 7)

(3, 6) → $f(x)$

domain: $2.75 \le x < \infty$

range: $-\infty < y \le 7$

(c) $x = 7 - \sqrt{4y-11}$, $x - 7 = -\sqrt{4y-11}$, $4y - 11 = (x-7)^2$,

$4y - 11 = x^2 - 14x + 49$, $4y = x^2 - 14x + 60$,

$y = f^{-1}(x) = 0.25x^2 - 3.5x + 15$ $\underbrace{(-\infty < x \le 7)}$

called the domain restriction