

4A.1. Angles on the unit circle

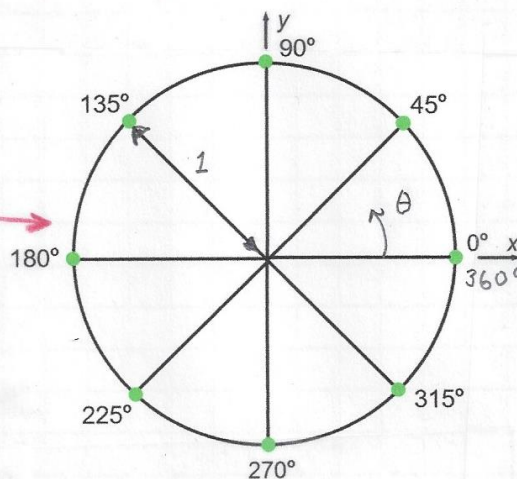
1 of 2

Angles in Standard Position $\equiv \theta$

- θ positive is measured counterclockwise from the positive x-axis
- θ negative " " clockwise

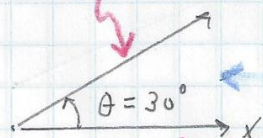
Note: There are 360° in a circle

Unit Circle



Coterminal Angles have the same initial and terminal sides.

terminal side



initial side

You can add or subtract revolutions (360°) to θ to describe the same angle. For example...

$\theta = 30^\circ + 360^\circ = 390^\circ$, $\theta = 30^\circ - 360^\circ = -330^\circ$
So, the θ -values -330° , 30° , 390° all define the angle at left. These three angles are coterminal.

Example #1. For each angle

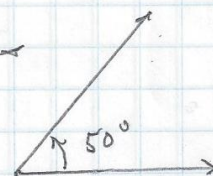
- Find two coterminal angles (one positive and one negative)
- Draw the angle

(a) $\theta = 770^\circ$

(b) $\theta = -250^\circ$

SOLUTION:

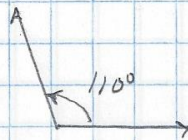
(a) $\theta = 770^\circ - 360^\circ = 410^\circ - 360^\circ = 50^\circ$ $- 360^\circ = -310^\circ$



4A.1, Angles on the Unit Circle

2 of 2

$$(b) \theta = -250^\circ + 360^\circ = 110^\circ \quad \theta = -250^\circ - 360^\circ = -610^\circ$$



Definition of Radians

The circumference of the unit circle is $(= \pi d = \pi(2)) = 2\pi$.

This is the definition of radians, i.e.

$$2\pi \text{ rad} = 360^\circ$$

or

$$\pi \text{ rad} = 180^\circ$$

You can leave off the "rad" with the understanding that the angle is in radians.

Example #2, Convert the angles to radians.

$$(a) \theta = 75^\circ \quad (b) \theta = -225^\circ$$

SOLUTION:

$$(a) \theta = 75^\circ = \left(\frac{75^\circ}{1}\right) \left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{12} \quad (b) \theta = -225^\circ = \left(\frac{-225^\circ}{1}\right) \left(\frac{\pi}{180^\circ}\right) = -\frac{5\pi}{4}$$

Example #3, Convert the angles to degrees.

$$(a) \theta = \frac{17\pi}{12} \quad (b) \theta = -\frac{15\pi}{6}$$

SOLUTION:

$$(a) \theta = \left(\frac{17\pi}{12}\right) \left(\frac{180^\circ}{\pi}\right) = 255^\circ \quad (b) \theta = \left(-\frac{15\pi}{6}\right) \left(\frac{180^\circ}{\pi}\right) = -450^\circ$$

Example #4, For each angle θ , find two coterminal angles in radians (one positive and one negative).

$$(a) \theta = \frac{13\pi}{5} \quad (b) \theta = -\frac{3\pi}{8}$$

SOLUTION:

$$(a) \theta = \frac{13\pi}{5} - 2\pi = \frac{3\pi}{5} \quad -2\pi = -\frac{7\pi}{5}$$

$$(b) \theta = -\frac{3\pi}{8} + 2\pi = \frac{13\pi}{8} \quad \theta = -\frac{3\pi}{8} - 2\pi = -\frac{19\pi}{8}$$