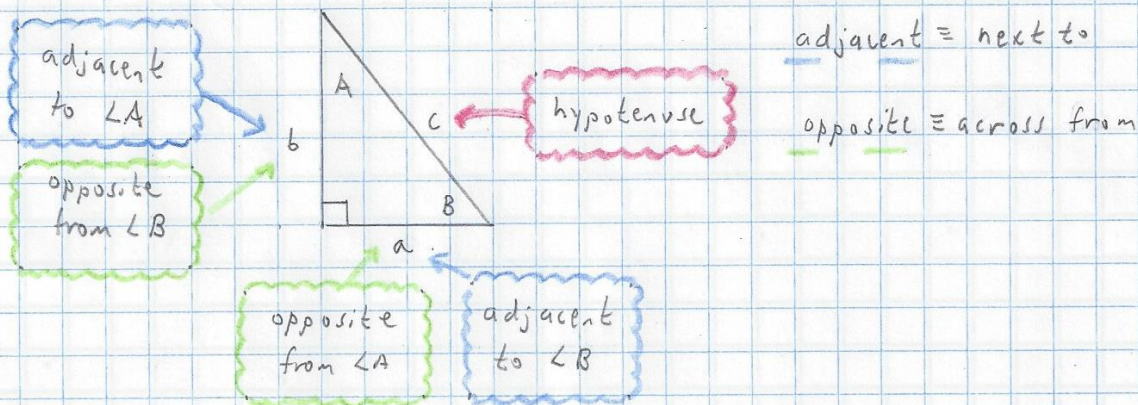


4A.5. The Sine, Cosine and Tangent Functions

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$$\sin = \frac{\text{opp}}{\text{hyp}} \quad \cos = \frac{\text{adj}}{\text{hyp}} \quad \tan = \frac{\text{opp}}{\text{adj}}$$

So, for example, for the triangle above...

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b} \quad \sin B = \frac{b}{c} \quad \cos B = \frac{a}{c} \quad \tan B = \frac{b}{a}$$

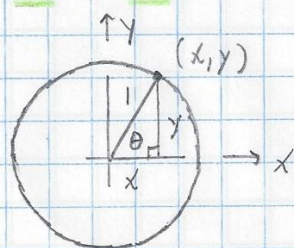
Note:

$$\frac{\sin}{\cos} = \frac{\frac{\text{opp}}{\text{hyp}}}{\frac{\text{adj}}{\text{hyp}}} = \frac{\text{opp}}{\text{adj}} = \tan$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$



Unit Circle



$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$$

So, on the unit circle

$$(x, y) = (\cos \theta, \sin \theta)$$

4A.5. The Sine, Cosine and Tangent Functions

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For Example #1 and #2, you can use the 4A.4 Unit Circle Notes, or you can use your calculator and memorize the decimal forms

$$\frac{1}{2} = 0.5, \quad \frac{\sqrt{2}}{2} = 0.7071, \quad \frac{\sqrt{3}}{2} = 0.8660$$

Example #1. Calculate exactly.

(a) $\cos 150^\circ$ (b) $\sin 510^\circ$ (c) $\cos(-120^\circ)$ (d) $\tan 30^\circ$

Solution:

(a) $\cos 150^\circ = -\frac{\sqrt{3}}{2}$ (b) $510^\circ - 360^\circ = 150^\circ$, $\sin 510^\circ = \sin 150^\circ = \frac{1}{2}$ \leftarrow

(c) $-120^\circ + 360^\circ = 240^\circ$, $\cos(-120^\circ) = \cos 240^\circ = -\frac{1}{2}$ \leftarrow

(d) $\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ \leftarrow

Example #2. Calculate exactly.

(a) $\sin \frac{9\pi}{4}$ (b) $\cos\left(-\frac{5\pi}{6}\right)$ (c) $\sin\left(-\frac{3\pi}{4}\right)$ (d) $\tan \frac{2\pi}{3}$

Solution:

(a) $\frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$, $\sin \frac{9\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ \leftarrow

(b) $-\frac{5\pi}{6} + 2\pi = \frac{7\pi}{6}$, $\cos\left(-\frac{5\pi}{6}\right) = \cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ \leftarrow

(c) $-\frac{3\pi}{4} + 2\pi = \frac{5\pi}{4}$, $\sin\left(-\frac{3\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ \leftarrow

(d) $\tan \frac{2\pi}{3} = \frac{\sin \frac{2\pi}{3}}{\cos \frac{2\pi}{3}} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$ \leftarrow