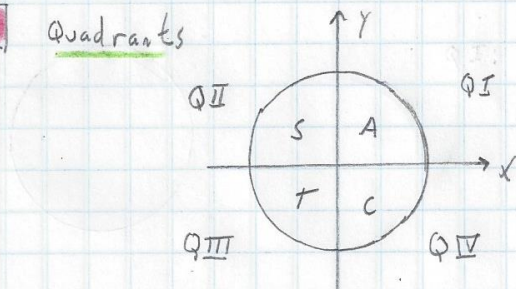


4A.6. Pythagorean Identity

1 of 2



Quadrants



$$\cos \theta = x$$

$$\sin \theta = y$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$$

ALL STUDENTS TAKE CALCULUS

QI: A (all, i.e., $\sin \theta$, $\cos \theta$ & $\tan \theta$, are positive)

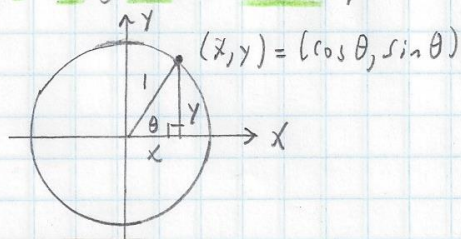
QII: S ($\sin \theta$ is positive)

QIII: T ($\tan \theta$ is positive)

QIV: C ($\cos \theta$ is positive)



Pythagorean Identity



Pythagorean Theorem $\Rightarrow x^2 + y^2 = 1^2 = 1$

$$\cos^2 \theta + \sin^2 \theta = 1$$

Example #1. For θ in Quadrant IV, $\cos \theta = \frac{12}{13}$. Find $\sin \theta$ and $\tan \theta$.

SOLUTION:

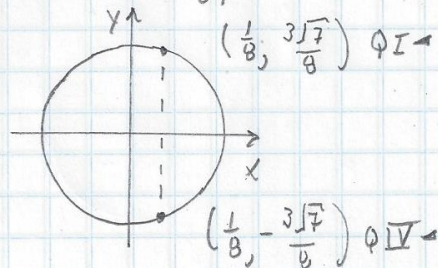
$$\cos^2 \theta + \sin^2 \theta = 1, \quad \sin^2 \theta = 1 - \cos^2 \theta = 1 - \left(\frac{12}{13}\right)^2 = \frac{25}{169},$$

$$QIV \Rightarrow \sin \theta = -\sqrt{\frac{25}{169}} = -\frac{5}{13} \quad \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-5/13}{12/13} = -\frac{5}{12}$$

Example #2. Two points $(x, y) = \left(\frac{1}{8}, y\right)$ are on the unit circle. Find the two values of y . In which quadrants are the two points?

SOLUTION: $x^2 + y^2 = 1, \quad \left(\frac{1}{8}\right)^2 + y^2 = 1, \quad y^2 = 1 - \left(\frac{1}{8}\right)^2 = \frac{63}{64}$

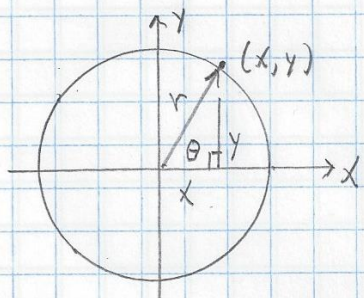
$$y = \pm \sqrt{\frac{63}{64}} = \pm \frac{\sqrt{9} \sqrt{7}}{8} = \pm \frac{3\sqrt{7}}{8} \approx \pm 0.9922$$



4A.6. Pythagorean Identity

2 of 2

Polar Coordinates (r, θ)



$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\cos \theta = \frac{x}{r}$$

$$\sin \theta = \frac{y}{r}$$

$$\tan \theta = \frac{y}{x}$$

Example #3. A point (x, y) on a circle of radius 7 is at angle $\theta = \frac{5\pi}{6}$. Find (x, y) .

SOLUTION:

$$\cos \theta = \frac{x}{r}, \quad x = r \cos \theta = 7 \cos \frac{5\pi}{6} = 7 \left(-\frac{\sqrt{3}}{2} \right) = -\frac{7\sqrt{3}}{2} \approx -6.0622$$

$$\sin \theta = \frac{y}{r}, \quad y = r \sin \theta = 7 \sin \frac{5\pi}{6} = 7 \left(\frac{1}{2} \right) = \frac{7}{2} = 3.5$$

Example #4. For θ in Quadrant III, $\tan \theta = \frac{4}{3}$, Find $\sin \theta$ and $\cos \theta$.

SOLUTION:

$$\text{Q III} \Rightarrow x < 0 \text{ and } y < 0, \quad \tan \theta = \frac{y}{x} = \frac{4}{3} \Rightarrow x = -3, \quad y = -4$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-4)^2} = 5$$

$$\sin \theta = \frac{y}{r} = \frac{-4}{5} \quad \cos \theta = \frac{x}{r} = \frac{-3}{5}$$