

## AP COMPUTER SCIENCE A – CONVERSION OF NUMBERS BETWEEN BASES

### Place Value.

The number 5768 in base 10, *i.e.*,  $5768_{10}$ , can be written as

$$5768_{10} = 10^3 \cdot 5 + 10^2 \cdot 7 + 10^1 \cdot 6 + 10^0 \cdot 8.$$

The place values of the digits are  $10^3 \cdots 10^0$ .

The number 377 in base 8, *i.e.*,  $377_8$ , can be written as

$$377_8 = 8^2 \cdot 3 + 8^1 \cdot 7 + 8^0 \cdot 7 = 255_{10}.$$

The place values of the digits are  $8^2 \cdots 8^0$ . In other words,  $377_8 = 255_{10}$ .

### Number of Digits.

The number of digits  $D_N$  of a number  $n$  in base  $N$  is

$$D_N = \text{ipart}(\log_N n) + 1,$$

where  $\text{ipart}$  is the integer part of  $\log_N n$ , *i.e.*, the fractional part is discarded. In Java, this equation can be coded as

```
int numDigits=(int) (Math.log((double)n)/Math.log((double)N))+1;
```

For example,  $603_{10}$  has, in base 5,  $D_5 = \text{ipart}(\log_5 603) + 1 = 4$  digits.

### Conversion of Base 10 to Base $N$ .

As an example, convert  $558_{10}$  to base 5. Then,  $D_5 = \text{ipart}(\log_5 558) + 1 = 4$ . So,  $558_{10} = abcd_5$ , or

$$558 = 5^3 \cdot a + 5^2 \cdot b + 5^1 \cdot c + 5^0 \cdot d = 125a + 25b + 5c + d.$$

$$\frac{558}{125} = 4.4 \dots \Rightarrow a = 4.$$

Subtracting  $125 \cdot 4$  from both sides gives

$$58 = 25b + 5c + d.$$

$$\frac{58}{25} = 2.3 \dots \Rightarrow b = 2.$$

Subtracting  $25 \cdot 2$  from both sides gives

$$8 = 5c + d.$$

$$\frac{8}{5} = 1.6 \dots \Rightarrow c = 1.$$

Subtracting  $5 \cdot 1$  from both sides gives

$$3 = d.$$

So,  $558_{10} = abcd_5 = 4213_5$ .