Section 2.1 Exercises

In Exercises 1–4, an object dropped from rest from the top of a tall building falls $y = 16t^2$ feet in the first t seconds.

1. Find the average speed during the first 3 seconds of fall.

2. Find the average speed during the first 4 seconds of fall.

3. Find the speed of the object at $t = 3$ seconds and confirm your answer algebraically.

4. Find the speed of the object at $t = 4$ seconds and confirm your answer algebraically.

In Exercises 5 and 6, use $\lim_{x\to c} k = k$, $\lim_{x\to c} x = c$, and the properties of limits to find the limit.

5. $\lim_{x\to c} (2x^3 - 3x^2 + x - 1)$

6. $\lim_{x\to c} \dfrac{x^4 - x^3 + 1}{x^2 + 9}$

In Exercises 7–14, determine the limit by substitution.

7. $\lim_{x\to -1/2} 3x^2(2x - 1)$

8. $\lim_{x\to -4} (x + 3)^{2016}$

9. $\lim_{x\to 1} (x^3 + 3x^2 - 2x - 17)$

10. $\lim_{y\to 2} \dfrac{y^2 + 5y + 6}{y + 2}$

11. $\lim_{y\to -3} \dfrac{y^2 + 4y + 3}{y^2 - 3}$

12. $\lim_{x\to 1/2} \operatorname{int} x$

13. $\lim_{x\to -2} (x - 6)^{2/3}$

14. $\lim_{x\to 2} \sqrt{x + 3}$

In Exercises 15–20, complete the following tables and state what you believe $\lim_{x\to 0} f(x)$ to be.

(a)

x	-0.1	-0.01	-0.001	-0.0001	...
$f(x)$	?	?	?	?	

(b)

x	0.1	0.01	0.001	0.0001	...
$f(x)$	?	?	?	?	

15. $f(x) = \dfrac{x^2 + 6x + 2}{x + 1}$

16. $f(x) = \dfrac{x^2 - x}{x}$

17. $f(x) = x \sin \dfrac{1}{x}$

18. $f(x) = \sin \dfrac{1}{x}$

19. $f(x) = \dfrac{10^x - 1}{x}$

20. $f(x) = x \sin(\ln |x|)$

In Exercises 21–24, explain why you cannot use substitution to determine the limit. Find the limit if it exists.

21. $\lim_{x\to -2} \sqrt{x - 2}$

22. $\lim_{x\to 0} \dfrac{1}{x^2}$

23. $\lim_{x\to 0} \dfrac{|x|}{x}$

24. $\lim_{x\to 0} \dfrac{(4 + x)^2 - 16}{x}$

In Exercises 25–34, explore the limit graphically. Confirm algebraically.

25. $\lim_{x\to 1} \dfrac{x - 1}{x^2 - 1}$

26. $\lim_{t\to 2} \dfrac{t^2 - 3t + 2}{t^2 - 4}$

27. $\lim_{x\to 0} \dfrac{5x^3 + 8x^2}{3x^4 - 16x^2}$

28. $\lim_{x\to 0} \dfrac{\frac{1}{2 + x} - \frac{1}{2}}{x}$

29. $\lim_{x\to 0} \dfrac{(2 + x)^3 - 8}{x}$

30. $\lim_{x\to 0} \dfrac{\sin 2x}{x}$

31. $\lim_{x\to 0} \dfrac{\sin x}{2x^2 - x}$

32. $\lim_{x\to 0} \dfrac{x + \sin x}{x}$

33. $\lim_{x\to 0} \dfrac{\sin^2 x}{x}$

34. $\lim_{x\to 5} \dfrac{x^3 - 125}{x - 5}$

In Exercises 35 and 36, use a graph to explore whether the limit exists.

35. $\lim_{x\to 1} \dfrac{x^2 - 4}{x - 1}$

36. $\lim_{x\to 2} \dfrac{x + 1}{x^2 - 4}$

In Exercises 37–42, determine the limit.

37. $\lim_{x\to 0^+} \operatorname{int} x$

38. $\lim_{x\to 0^-} \operatorname{int} x$

39. $\lim_{x\to 0.01} \operatorname{int} x$

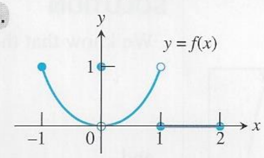
40. $\lim_{x\to 2^-} \operatorname{int} x$

41. $\lim_{x\to 0^+} \dfrac{x}{|x|}$

42. $\lim_{x\to 0^-} \dfrac{x}{|x|}$

In Exercises 43 and 44, which of the statements are true about the function $y = f(x)$ graphed there, and which are false?

43.



(a) $\lim_{x\to -1^+} f(x) = 1$

(b) $\lim_{x\to 0^-} f(x) = 0$

(c) $\lim_{x\to 0^-} f(x) = 1$

(d) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

(e) $\lim_{x\to 0} f(x)$ exists

(f) $\lim_{x\to 0} f(x) = 0$

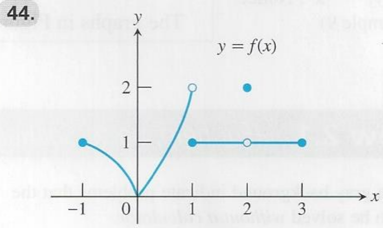
(g) $\lim_{x\to 0} f(x) = 1$

(h) $\lim_{x\to 1} f(x) = 1$

(i) $\lim_{x\to 1} f(x) = 0$

(j) $\lim_{x\to 2^-} f(x) = 2$

44.



(a) $\lim_{x\to -1^+} f(x) = 1$

(b) $\lim_{x\to 2} f(x)$ does not exist.

(c) $\lim_{x\to 2} f(x) = 2$

(d) $\lim_{x\to 1^-} f(x) = 2$

(e) $\lim_{x\to 1^+} f(x) = 1$

(f) $\lim_{x\to 1} f(x)$ does not exist.

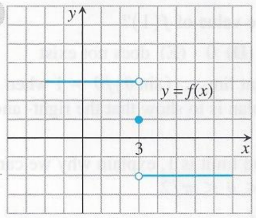
(g) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

(h) $\lim_{x\to c} f(x)$ exists at every c in $(-1, 1)$.

(i) $\lim_{x\to c} f(x)$ exists at every c in $(1, 3)$.

In Exercises 45–50, use the graph to estimate the limits and value of the function, or explain why the limits do not exist.

45.



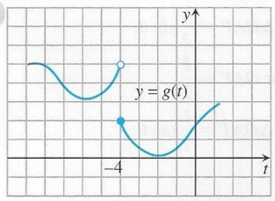
(a) $\lim_{x\to 3^-} f(x)$

(b) $\lim_{x\to 3^+} f(x)$

(c) $\lim_{x\to 3} f(x)$

(d) $f(3)$

46.



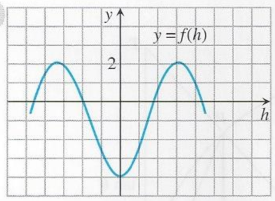
(a) $\lim_{t\to -4^-} g(t)$

(b) $\lim_{t\to -4^+} g(t)$

(c) $\lim_{t\to -4} g(t)$

(d) $g(-4)$

47.



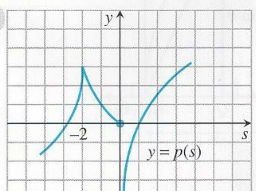
(a) $\lim_{h\to 0^-} f(h)$

(b) $\lim_{h\to 0^+} f(h)$

(c) $\lim_{h\to 0} f(h)$

(d) $f(0)$

48.



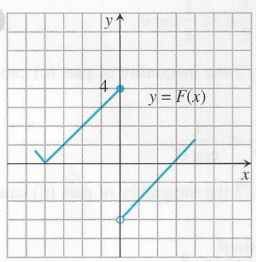
(a) $\lim_{s\to -2^-} p(s)$

(b) $\lim_{s\to -2^+} p(s)$

(c) $\lim_{s\to -2} p(s)$

(d) $p(-2)$

49.



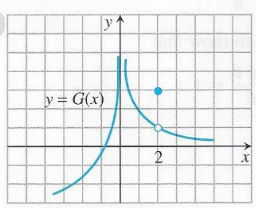
(a) $\lim_{x\to 0^-} F(x)$

(b) $\lim_{x\to 0^+} F(x)$

(c) $\lim_{x\to 0} F(x)$

(d) $F(0)$

50.



(a) $\lim_{x\to 2^-} G(x)$

(b) $\lim_{x\to 2^+} G(x)$

(c) $\lim_{x\to 2} G(x)$

(d) $G(2)$

In Exercises 51–54, match the function with the table.

51. $y_1 = \dfrac{x^2 + x - 2}{x - 1}$

52. $y_1 = \dfrac{x^2 - x - 2}{x - 1}$

53. $y_1 = \dfrac{x^2 - 2x + 1}{x - 1}$

54. $y_1 = \dfrac{x^2 + x - 2}{x + 1}$

X	Y1
.7	−.4765
.8	−.3111
.9	−.1526
1	0
1.1	.14762
1.2	.29091
1.3	.43043
X = .7	

(a)

X	Y1
.7	7.3667
.8	10.8
.9	20.9
1	ERROR
1.1	−18.9
1.2	−8.8
1.3	−5.367
X = .7	

(b)

X	Y1
.7	2.7
.8	2.8
.9	2.9
1	ERROR
1.1	3.1
1.2	3.2
1.3	3.3
X = .7	

(c)

X	Y1
.7	−.3
.8	−.2
.9	−.1
1	ERROR
1.1	.1
1.2	.2
1.3	.3
X = .7	

(d)

In Exercises 55 and 56, determine the limit.

55. Assume that $\lim_{x\to 4} f(x) = 0$ and $\lim_{x\to 4} g(x) = 3$.

(a) $\lim_{x\to 4} (g(x) + 3)$

(b) $\lim_{x\to 4} x f(x)$

(c) $\lim_{x\to 4} g^2(x)$

(d) $\lim_{x\to 4} \dfrac{g(x)}{f(x) - 1}$

56. Assume that $\lim_{x\to b} f(x) = 7$ and $\lim_{x\to b} g(x) = -3$.

(a) $\lim_{x\to b} (f(x) + g(x))$

(b) $\lim_{x\to b} (f(x) \cdot g(x))$

(c) $\lim_{x\to b} 4\, g(x)$

(d) $\lim_{x\to b} \dfrac{f(x)}{g(x)}$

In Exercises 57–60, complete parts (a), (b), and (c) for the piecewise-defined function.

(a) Draw the graph of f.

(b) Determine $\lim_{x\to c^+} f(x)$ and $\lim_{x\to c^-} f(x)$.

(c) **Writing to Learn** Does $\lim_{x\to c} f(x)$ exist? If so, what is it? If not, explain.

57. $c = 2,\ f(x) = \begin{cases} 3 - x, & x < 2 \\ \dfrac{x}{2} + 1, & x > 2 \end{cases}$

58. $c = 2,\ f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ x/2, & x > 2 \end{cases}$

59. $c = 1,\ f(x) = \begin{cases} \dfrac{1}{x - 1}, & x < 1 \\ x^3 - 2x + 5, & x \ge 1 \end{cases}$

60. $c = -1,\ f(x) = \begin{cases} 1 - x^2, & x \ne -1 \\ 2, & x = -1 \end{cases}$

In Exercises 61–64, complete parts (a)–(d) for the piecewise-defined function.

(a) Draw the graph of f.

(b) At what points c in the domain of f does $\lim_{x\to c} f(x)$ exist?

(c) At what points c does only the left-hand limit exist?

(d) At what points c does only the right-hand limit exist?

61. $f(x) = \begin{cases} \sin x, & -2\pi \le x < 0 \\ \cos x, & 0 \le x \le 2\pi \end{cases}$

62. $f(x) = \begin{cases} \cos x, & -\pi \le x < 0 \\ \sec x, & 0 \le x \le \pi \end{cases}$

63. $f(x) = \begin{cases} \sqrt{1-x^2}, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \end{cases}$

64. $f(x) = \begin{cases} x, & -1 \le x < 0, \text{ or } 0 < x \le 1 \\ 1, & x = 0 \\ 0, & x < -1, \text{ or } x > 1 \end{cases}$

In Exercises 65–68, find the limit graphically. Use the Squeeze Theorem to confirm your answer.

65. $\lim_{x\to 0} x \sin x$

66. $\lim_{x\to 0} x^2 \sin x$

67. $\lim_{x\to 0} x^2 \sin \dfrac{1}{x^2}$

68. $\lim_{x\to 0} x^2 \cos \dfrac{1}{x^2}$

69. ***Free Fall*** A water balloon dropped from a window high above the ground falls $y = 4.9t^2$ m in t sec. Find the balloon's

(a) average speed during the first 3 sec of fall.

(b) speed at the instant $t = 3$.

70. ***Free Fall on a Small Airless Planet*** A rock released from rest to fall on a small airless planet falls $y = gt^2$ m in t sec, g a constant. Suppose that the rock falls to the bottom of a crevasse 20 m below and reaches the bottom in 4 sec.

(a) Find the value of g.

(b) Find the average speed for the fall.

(c) With what speed did the rock hit the bottom?

Standardized Test Questions

71. True or False If $\lim_{x\to c^-} f(x) = 2$ and $\lim_{x\to c^+} f(x) = 2$, then $\lim_{x\to c} f(x) = 2$. Justify your answer.

72. True or False $\lim_{x\to 0} \dfrac{x + \sin x}{x} = 2$. Justify your answer.

In Exercises 73–76, use the following function.

$$f(x) = \begin{cases} 2 - x, & x \le 1 \\ \dfrac{x}{2} + 1, & x > 1 \end{cases}$$

73. Multiple Choice What is the value of $\lim_{x\to 1^-} f(x)$?

(A) $5/2$ **(B)** $3/2$ **(C)** 1 **(D)** 0 **(E)** does not exist

74. Multiple Choice What is the value of $\lim_{x\to 1^+} f(x)$?

(A) $5/2$ **(B)** $3/2$ **(C)** 1 **(D)** 0 **(E)** does not exist

75. Multiple Choice What is the value of $\lim_{x\to 1} f(x)$?

(A) $5/2$ **(B)** $3/2$ **(C)** 1 **(D)** 0 **(E)** does not exist

76. Multiple Choice What is the value of $f(1)$?

(A) $5/2$ **(B)** $3/2$ **(C)** 1 **(D)** 0 **(E)** does not exist

77. Group Activity To prove that $\lim_{\theta\to 0} (\sin\theta)/\theta = 1$ when θ is measured in radians, the plan is to show that the right- and left-hand limits are both 1.

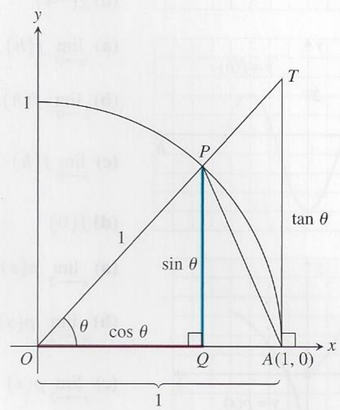
(a) To show that the right-hand limit is 1, explain why we can restrict our attention to $0 < \theta < \pi/2$.

(b) Use the figure to show that

$$\text{area of } \Delta OAP = \frac{1}{2}\sin\theta,$$

$$\text{area of sector } OAP = \frac{\theta}{2},$$

$$\text{area of } \Delta OAT = \frac{1}{2}\tan\theta.$$



(c) Use part (b) and the figure to show that for $0 < \theta < \pi/2$,

$$\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{1}{2}\tan\theta.$$

(d) Show that for $0 < \theta < \pi/2$ the inequality of part (c) can be written in the form

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}.$$

(e) Show that for $0 < \theta < \pi/2$ the inequality of part (d) can be written in the form

$$\cos\theta < \frac{\sin\theta}{\theta} < 1.$$

(f) Use the Squeeze Theorem to show that

$$\lim_{\theta\to 0^+} \frac{\sin\theta}{\theta} = 1.$$

(g) Show that $(\sin\theta)/\theta$ is an even function.

(h) Use part (g) to show that

$$\lim_{\theta\to 0^-}\frac{\sin\theta}{\theta} = 1.$$

(i) Finally, show that

$$\lim_{\theta\to 0^+}\frac{\sin\theta}{\theta} = 1.$$

Extending the Ideas

78. *Controlling Outputs* Let $f(x) = \sqrt{3x - 2}$.

(a) Show that $\lim_{x\to 2} f(x) = 2 = f(2)$.

(b) Use a graph to estimate values for a and b so that $1.8 < f(x) < 2.2$ provided $a < x < b$.

(c) Use a graph to estimate values for a and b so that $1.99 < f(x) < 2.01$ provided $a < x < b$.

79. *Controlling Outputs* Let $f(x) = \sin x$.

(a) Find $f(\pi/6)$.

(b) Use a graph to estimate an interval (a, b) about $x = \pi/6$ so that $0.3 < f(x) < 0.7$ provided $a < x < b$.

(c) Use a graph to estimate an interval (a, b) about $x = \pi/6$ so that $0.49 < f(x) < 0.51$ provided $a < x < b$.

80. *Limits and Geometry* Let $P(a, a^2)$ be a point on the parabola $y = x^2$, $a > 0$. Let O be the origin and $(0, b)$ the y-intercept of the perpendicular bisector of line segment OP. Find $\lim_{P\to O} b$.