Section 2.3 Exercises

In Exercises 1–10, find the points of continuity and the points of discontinuity of the function. Identify each type of discontinuity.

1. $y = \dfrac{1}{(x+2)^2}$

2. $y = \dfrac{x+1}{x^2-4x+3}$

3. $y = \dfrac{1}{x^2+1}$

4. $y = |x-1|$

5. $y = \sqrt{2x+3}$

6. $y = \sqrt[3]{2x-1}$

7. $y = |x|/x$

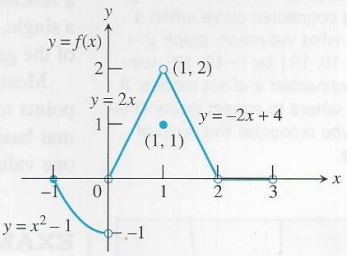
8. $y = \cot x$

9. $y = e^{1/x}$

10. $y = \ln(x+1)$

In Exercises 11–18, use the function f defined and graphed below to answer the questions.

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



11. **(a)** Does $f(-1)$ exist?

(b) Does $\lim_{x \to -1^+} f(x)$ exist?

(c) Does $\lim_{x \to -1^+} f(x) = f(-1)$?

(d) Is f continuous at $x = -1$?

12. **(a)** Does $f(1)$ exist?

(b) Does $\lim_{x \to 1} f(x)$ exist?

(c) Does $\lim_{x \to 1} f(x) = f(1)$?

(d) Is f continuous at $x = 1$?

13. **(a)** Is f defined at $x = 2$? (Look at the definition of f.)
(b) Is f continuous at $x = 2$?

14. At what values of x is f continuous?

15. What value should be assigned to $f(2)$ to make the extended function continuous at $x = 2$?

16. What new value should be assigned to $f(1)$ to make the new function continuous at $x = 1$?

17. Writing to Learn Is it possible to extend f to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

18. Writing to Learn Is it possible to extend f to be continuous at $x = 3$? If so, what value should the extended function have there? If not, why not?

In Exercises 19–24, **(a)** find each point of discontinuity. **(b)** Which of the discontinuities are removable? not removable? Give reasons for your answers.

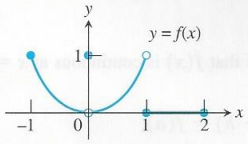
19. $f(x) = \begin{cases} 3 - x, & x < 2 \\ \dfrac{x}{2} + 1, & x > 2 \end{cases}$

20. $f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ x/2, & x > 2 \end{cases}$

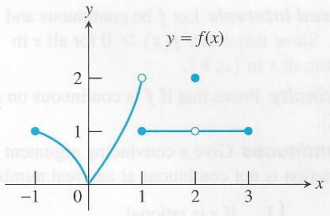
21. $f(x) = \begin{cases} \dfrac{1}{x - 1}, & x < 1 \\ x^3 - 2x + 5, & x \geq 1 \end{cases}$

22. $f(x) = \begin{cases} 1 - x^2, & x \neq -1 \\ 2, & x = -1 \end{cases}$

23.



24.



In Exercises 25–30, give a formula for the extended function that is continuous at the indicated point.

25. $f(x) = \dfrac{x^2 - 9}{x + 3}, \quad x = -3$

26. $f(x) = \dfrac{x^3 - 1}{x^2 - 1}, \quad x = 1$

27. $f(x) = \dfrac{\sin x}{x}, \quad x = 0$

28. $f(x) = \dfrac{\sin 4x}{x}, \quad x = 0$

29. $f(x) = \dfrac{x - 4}{\sqrt{x} - 2}, \quad x = 4$

30. $f(x) = \dfrac{x^3 - 4x^2 - 11x + 30}{x^2 - 4}, \quad x = 2$

In Exercises 31 and 32, explain why the given function is continuous.

31. $f(x) = \dfrac{1}{x - 3}$

32. $g(x) = \dfrac{1}{\sqrt{x - 1}}$

In Exercises 33–36, use Theorem 7 to show that the given function is continuous.

33. $f(x) = \sqrt{\left(\dfrac{x}{x + 1}\right)}$

34. $f(x) = \sin(x^2 + 1)$

35. $f(x) = \cos\left(\sqrt[3]{1 - x}\right)$

36. $f(x) = \tan\left(\dfrac{x^2}{x^2 + 4}\right)$

Group Activity In Exercises 37–40, verify that the function is continuous and state its domain. Indicate which theorems you are using, and which functions you are assuming to be continuous.

37. $y = \dfrac{1}{\sqrt{x + 2}}$

38. $y = x^2 + \sqrt[3]{4 - x}$

39. $y = |x^2 - 4x|$

40. $y = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$

In Exercises 41–44, sketch a possible graph for a function f that has the stated properties.

41. $f(3)$ exists but $\lim_{x \to 3} f(x)$ does not.

42. $f(-2)$ exists, $\lim_{x \to -2^+} f(x) = f(-2)$, but $\lim_{x \to -2} f(x)$ does not exist.

43. $f(4)$ exists, $\lim_{x \to 4} f(x)$ exists, but f is not continuous at $x = 4$.

44. $f(x)$ is continuous for all x except $x = 1$, where f has a nonremovable discontinuity.

45. *Solving Equations* Is any real number exactly 1 less than its fourth power? Give any such values accurate to 3 decimal places.

46. *Solving Equations* Is any real number exactly 2 more than its cube? Give any such values accurate to 3 decimal places.

47. *Continuous Function* Find a value for a so that the function

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

is continuous.

48. *Continuous Function* Find a value for a so that the function

$$f(x) = \begin{cases} 2x + 3, & x \leq 2 \\ ax + 1, & x > 2 \end{cases}$$

is continuous.

49. *Continuous Function* Find a value for a so that the function

$$f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$$

is continuous.

50. *Continuous Function* Find a value for a so that the function

$$f(x) = \begin{cases} x^2 + x + a, & x < 1 \\ x^3, & x \geq 1 \end{cases}$$

is continuous.

51. Writing to Learn Explain why the equation $e^{-x} = x$ has at least one solution.

52. *Salary Negotiation* A welder's contract promises a 3.5% salary increase each year for 4 years and Luisa has an initial salary of $36,500.

(a) Show that Luisa's salary is given by

$$y = 36{,}500(1.035)^{\text{int } t},$$

where t is the time, measured in years, since Luisa signed the contract.

(b) Graph Luisa's salary function. At what values of t is it continuous?

53. *Airport Parking* Valuepark charges $1.10 per hour or fraction of an hour for airport parking. The maximum charge per day is $7.25.

(a) Write a formula that gives the charge for x hours with $0 \leq x \leq 24$. (*Hint:* See Exercise 52.)

(b) Graph the function in part (a). At what values of x is it continuous?

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

54. True or False A continuous function cannot have a point of discontinuity. Justify your answer.

55. True or False It is possible to extend the definition of a function f at a jump discontinuity $x = a$ so that f is continuous at $x = a$. Justify your answer.

56. Multiple Choice On which of the following intervals is $f(x) = \dfrac{1}{\sqrt{x}}$ not continuous?

(A) $(0, \infty)$ **(B)** $[0, \infty)$ **(C)** $(0, 2)$
(D) $(1, 2)$ **(E)** $[1, \infty)$

57. Multiple Choice Which of the following points is not a point of discontinuity of $f(x) = \sqrt{x - 1}$?

(A) $x = -1$ **(B)** $x = -1/2$ **(C)** $x = 0$
(D) $x = 1/2$ **(E)** $x = 1$

58. Multiple Choice Which of the following statements about the function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -x + 3, & 1 < x < 2 \end{cases}$$

is not true?

(A) $f(1)$ does not exist.
(B) $\lim_{x \to 0^+} f(x)$ exists.
(C) $\lim_{x \to 2^-} f(x)$ exists.
(D) $\lim_{x \to 1} f(x)$ exists.
(E) $\lim_{x \to 1} f(x) \neq f(1)$

59. Multiple Choice Which of the following points of discontinuity of

$$f(x) = \frac{x(x-1)(x-2)^2(x+1)^2(x-3)^2}{x(x-1)(x-2)(x+1)^2(x-3)^3}$$

is not removable?

(A) $x = -1$ **(B)** $x = 0$ **(C)** $x = 1$
(D) $x = 2$ **(E)** $x = 3$

Exploration

60. Let $f(x) = \left(1 + \dfrac{1}{x}\right)^x$.

(a) Find the domain of f. **(b)** Draw the graph of f.

(c) Writing to Learn Explain why $x = -1$ and $x = 0$ are points of discontinuity of f.

(d) Writing to Learn Is either of the discontinuities in part (c) removable? Explain.

(e) Use graphs and tables to estimate $\lim_{x \to \infty} f(x)$.

Extending the Ideas

61. *Continuity at a Point* Show that $f(x)$ is continuous at $x = a$ if and only if

$$\lim_{h \to 0} f(a + h) = f(a).$$

62. *Continuity on Closed Intervals* Let f be continuous and never zero on $[a, b]$. Show that either $f(x) > 0$ for all x in $[a, b]$ or $f(x) < 0$ for all x in $[a, b]$.

63. *Properties of Continuity* Prove that if f is continuous on an interval, then so is $|f|$.

64. *Everywhere Discontinuous* Give a convincing argument that the following function is not continuous at any real number.

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$