

Section 2.4 Exercises

In Exercises 1–6, find the average rate of change of the function over each interval.

1. $f(x) = x^3 + 1$

(a) $[2, 3]$ (b) $[-1, 1]$

2. $f(x) = \sqrt{4x + 1}$

(a) $[0, 2]$ (b) $[10, 12]$

3. $f(x) = e^x$

(a) $[-2, 0]$ (b) $[1, 3]$

4. $f(x) = \ln x$

(a) $[1, 4]$ (b) $[100, 103]$

5. $f(x) = \cot x$

(a) $[\pi/4, 3\pi/4]$ (b) $[\pi/6, \pi/2]$

6. $f(x) = 2 + \cos x$

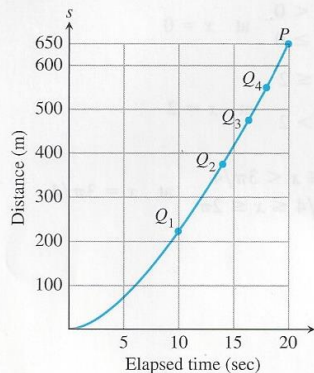
(a) $[0, \pi]$ (b) $[-\pi, \pi]$

In Exercises 7 and 8, a distance-time graph is shown.

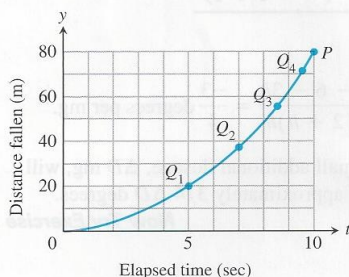
- (a) Estimate the slopes of the secants PQ_1 , PQ_2 , PQ_3 , and PQ_4 , arranging them in order in a table. What is the appropriate unit for these slopes?

- (b) Estimate the speed at point P .

7. **Accelerating from a Standstill** The figure shows the distance-time graph for a 1994 Ford® Mustang Cobra™ accelerating from a standstill.



- 8. Lunar Data** The accompanying figure shows a distance-time graph for a wrench that fell from the top platform of a communication mast on the moon to the station roof 80 m below.



In Exercises 9–12, at the indicated point find

- the slope of the curve,
- an equation of the tangent, and
- an equation of the normal.
- Then draw a graph of the curve, tangent line, and normal line in the same square viewing window.

9. $y = x^2$ at $x = -2$ 10. $y = x^2 - 4x$ at $x = 1$
 11. $y = \frac{1}{x-1}$ at $x = 2$ 12. $y = x^2 - 3x - 1$ at $x = 0$

In Exercises 13 and 14, find the slope of the curve at the indicated point.

13. $f(x) = |x|$ at (a) $x = 2$ (b) $x = -3$
 14. $f(x) = |x - 2|$ at $x = 1$

In Exercises 15–18, determine whether the curve has a tangent at the indicated point. If it does, give its slope. If not, explain why not.

15. $f(x) = \begin{cases} 2 - 2x - x^2, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$ at $x = 0$
 16. $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - x, & x \geq 0 \end{cases}$ at $x = 0$
 17. $f(x) = \begin{cases} 1/x, & x \leq 2 \\ \frac{4-x}{4}, & x > 2 \end{cases}$ at $x = 2$
 18. $f(x) = \begin{cases} \sin x, & 0 \leq x < 3\pi/4 \\ \cos x, & 3\pi/4 \leq x \leq 2\pi \end{cases}$ at $x = 3\pi/4$

In Exercises 19–22, (a) find the slope of the curve at $x = a$.

- (b) **Writing to Learn** Describe what happens to the tangent at $x = a$ as a changes.

19. $y = x^2 + 2$

20. $y = 2/x$

21. $y = \frac{1}{x-1}$

22. $y = 9 - x^2$

Find the instantaneous rate of change of the position function $y = f(t)$ in feet at the given time t in seconds.

23. $f(t) = 3t - 7$, $t = 1$

24. $f(t) = 3t^2 + 2t$, $t = 3$

25. $f(t) = \frac{t+1}{t}$, $t = 2$

26. $f(t) = t^3 - 1$, $t = 2$

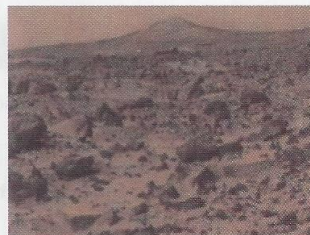
27. **Free Fall** An object is dropped from the top of a 100-m tower. Its height above ground after t sec is $100 - 4.9t^2$ m. How fast is it falling 2 sec after it is dropped?

28. **Rocket Launch** At t sec after lift-off, the height of a rocket is $3t^2$ ft. How fast is the rocket climbing after 10 sec?

29. **Area of Circle** What is the rate of change of the area of a circle with respect to the radius when the radius is $r = 3$ in.?

30. **Volume of Sphere** What is the rate of change of the volume of a sphere with respect to the radius when the radius is $r = 2$ in.?

31. **Free Fall on Mars** The equation for free fall at the surface of Mars is $s = 1.86t^2$ m with t in seconds. Assume a rock is dropped from the top of a 200-m cliff. Find the speed of the rock at $t = 1$ sec.



32. **Free Fall on Jupiter** The equation for free fall at the surface of Jupiter is $s = 11.44t^2$ m with t in seconds. Assume a rock is dropped from the top of a 500-m cliff. Find the speed of the rock at $t = 2$ sec.

33. **Horizontal Tangent** At what point is the tangent to $f(x) = x^2 + 4x - 1$ horizontal?

34. **Horizontal Tangent** At what point is the tangent to $f(x) = 3 - 4x - x^2$ horizontal?

35. Finding Tangents and Normals

- (a) Find an equation for each tangent to the curve $y = 1/(x - 1)$ that has slope -1 . (See Exercise 21.)
- (b) Find an equation for each normal to the curve $y = 1/(x - 1)$ that has slope 1.

36. Finding Tangents Find the equations of all lines tangent to $y = 9 - x^2$ that pass through the point $(1, 12)$.

37. Sensitivity A patient's temperature T as a function of the dosage D of a medicine is given by $T(D) = 99 + 4/(1 + D)$. Find and interpret the sensitivity of the patient's temperature to the dosage when $D = 2$ mg.

38. Sensitivity If a ball is thrown straight up with an initial velocity of v feet per second, it will reach a maximum height of $H = v^2/64$ feet. Find and interpret the sensitivity of the height to the initial velocity when the initial velocity is 40 ft/sec.

39. Table 2.2 gives the total amount of all U.S. exported wheat in millions of bushels for several years.

TABLE 2.2 U.S. Exported Wheat

Year	Exported Wheat (millions of bushels)
2008	1015
2009	879
2010	1291
2011	1051
2012	1007
2013	900

Source: U.S. Department of Agriculture, Economic Research Service, Wheat Data, Table 21.

- (a) Make a scatter plot of the data in the table.
- (b) Let P represent the point corresponding to 2008, Q_1 the point corresponding to 2011, Q_2 the point corresponding to 2012, and Q_3 the point corresponding to 2013. Find the slope of the secant line PQ_i for $i = 1, 2, 3$.
- 40.** Table 2.3 gives the amount of federal spending in billions of dollars for national defense for several years.

TABLE 2.3 National Defense Spending

Year	National Defense Spending (\$ billion)
2008	616
2009	661
2010	693
2011	706
2012	678
2013	633

Source: U.S. Office of Management and Budget, Budget Authority by Function and Subfunction, Outlay by Function and Subfunction, Table 492.

- (a) Find the average rate of change in spending from 2008 to 2013.
- (b) Find the average rate of change in spending from 2008 to 2011.
- (c) Find the average rate of change in spending from 2011 to 2013.
- (d) **Writing to Learn** Explain why someone might be hesitant to make predictions about the rate of change of national defense spending based on the data given in Table 2.3.

Standardized Test Questions

- 41. True or False** If the graph of a function has a tangent line at $x = a$, then the graph also has a normal line at $x = a$. Justify your answer.
- 42. True or False** The graph of $f(x) = |x|$ has a tangent line at $x = 0$. Justify your answer.
- 43. Multiple Choice** If the line L tangent to the graph of a function f at the point $(2, 5)$ passes through the point $(-1, -3)$, what is the slope of L ?
- (A) $-3/8$ (B) $3/8$ (C) $-8/3$ (D) $8/3$ (E) undefined
- 44. Multiple Choice** Find the average rate of change of $f(x) = x^2 + x$ over the interval $[1, 3]$.
- (A) -5 (B) $1/5$ (C) $1/4$ (D) 4 (E) 5
- 45. Multiple Choice** Which of the following is an equation of the tangent to the graph of $f(x) = 2/x$ at $x = 1$?
- (A) $y = -2x$ (B) $y = 2x$ (C) $y = -2x + 4$
- (D) $y = -x + 3$ (E) $y = x + 3$
- 46. Multiple Choice** Which of the following is an equation of the normal to the graph of $f(x) = 2/x$ at $x = 1$?
- (A) $y = \frac{1}{2}x + \frac{3}{2}$ (B) $y = -\frac{1}{2}x$ (C) $y = \frac{1}{2}x + 2$
- (D) $y = -\frac{1}{2}x + 2$ (E) $y = 2x + 5$

Explorations

In Exercises 47 and 48, complete the following for the function.

- (a) Compute the difference quotient

$$\frac{f(1+h) - f(1)}{h}$$

- (b) Use graphs and tables to estimate the limit of the difference quotient in part (a) as $h \rightarrow 0$.
- (c) Compare your estimate in part (b) with the given number.
- (d) **Writing to Learn** Based on your computations, do you think the graph of f has a tangent at $x = 1$? If so, estimate its slope. If not, explain why not.

- 47.** $f(x) = e^x$, e **48.** $f(x) = 2^x$, $\ln 4$

Group Activity In Exercises 49–52, the curve $y = f(x)$ has a **vertical tangent** at $x = a$ if

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \infty$$

or if

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = -\infty.$$

In each case, the right- and left-hand limits are required to be the same: both $+\infty$ or both $-\infty$.

Use graphs to investigate whether the curve has a vertical tangent at $x = 0$.

49. $y = x^{2/5}$

50. $y = x^{3/5}$

51. $y = x^{1/3}$

52. $y = x^{2/3}$

Extending the Ideas

In Exercises 53 and 54, determine whether the graph of the function has a tangent at the origin. Explain your answer.

$$53. f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$54. f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

55. **Sine Function** Estimate the slope of the curve $y = \sin x$ at $x = 1$. (Hint: See Exercises 47 and 48.)

56. Consider the function f given in Example 1. Explain how the average rate of change of f over the interval $[3, 3+h]$ is the same as the difference quotient of f at $a = 3$.

57. (a) Let $x = a + h$. Show algebraically how the difference quotient of f at a ,

$$\frac{f(a+h) - f(a)}{h},$$

is equivalent to an alternate form given by

$$\frac{f(x) - f(a)}{x - a}.$$

(b) **Writing to Learn** Why do you think we discuss two forms of the difference quotient of f at a ?