2.2. End-Behavior Functions of Rational Functions

Example #1. For $f(x) = \dfrac{x^2-2x-15}{x-1} = \dfrac{P(x)}{Q(x)}$,

(a) Calculate the end-behavior function $y = E(x)$.

(b) Find the vertical asymptote of $y = f(x)$.

(c) Graph $y = f(x)$ and $y = E(x)$ with your calculator.

Window: $x \in [-15, 15]$ & $y \in [-15, 15]$

SOLUTION:

(a)

1	1	-2	-15
		1	-1
	1	-1	-16

$f(x) = x - 1 - \dfrac{16}{x-1}$ $\Rightarrow E(x) = x - 1$

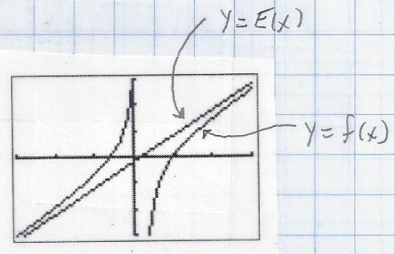
$\dfrac{16}{x-1} \to 0$ as $x \to \pm\infty$

(b) $P(1) = -16 \Rightarrow x = 1$ is V.A.

(c)

WINDOW
Xmin=-15
Xmax=15
Xscl=5
Ymin=-15
Ymax=15
Yscl=5
↓Xres=1

Plot1 Plot2 Plot3
\Y1■(X^2-2X-15)/(X-1)
\Y2■X-1
\Y3=
\Y4=
\Y5=
\Y6=



Comments:

1) $y = f(x)$ looks like $y = E(x)$ for large x

2) $y = E(x)$ in this case is an oblique asymptote

Example #2. For $f(x) = \dfrac{x^3-x^2-17x-15}{x-1} = \dfrac{P(x)}{Q(x)}$,

(a) Calculate the end-behavior function $y = E(x)$.

(b) Find the vertical asymptote of $y = f(x)$.

(c) Graph $y = f(x)$ and $y = E(x)$ with your calculator.

window: $x \in [-10, 10]$ and $y \in [-50, 50]$.

SOLUTION:

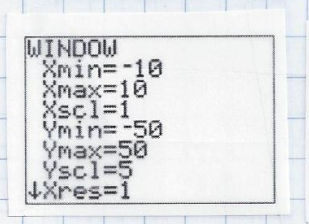
(a)

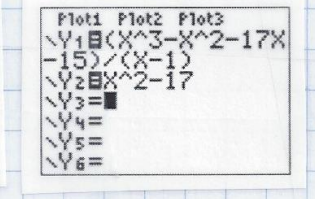
1	1	-1	-17	-15
		1	0	-17
	1	0	-17	-32

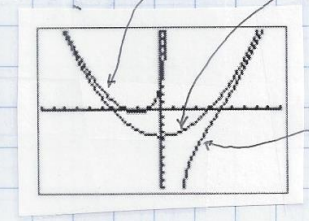
$f(x) = x^2 - 17 - \underbrace{\frac{32}{x-1}}_{\to 0 \text{ as } x \to \pm\infty}$ $\Rightarrow E(x) = x^2 - 17$ ◄

(b) $P(1) = -32 \Rightarrow x = 1$ is V.A. ◄

(c)







Note: $y = f(x)$ looks like $y = E(x)$ for large x.