HW #1

2.1 Definition of Limits

pg. 65

(25) $\lim_{x\to 1} \dfrac{x-1}{x^2-1} = \lim_{x\to 1} \dfrac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \lim_{x\to 1} \dfrac{1}{x+1} = \dfrac{1}{1+1} = \dfrac{1}{2}$

(29) $\lim_{x\to 0} \dfrac{(2+x)^3-8}{x} = \lim_{x\to 0} \dfrac{(8+12x+6x^2+x^3)-8}{x} = \lim_{x\to 0} \dfrac{12x+6x^2+x^3}{x} =$

$= \lim_{x\to 0} (12+6x+x^2) = 12$

(34)

$$\begin{array}{r|rrr|r} 5 & 1 & 0 & 0 & -125 \\ & & 5 & 25 & 125 \\ \hline & 1 & 5 & 25 & 0 \end{array}$$

$\lim_{x\to 5} \dfrac{x^3-125}{x-5} = \lim_{x\to 5} (x^2+5x+25) = 5^2+5(5)+25 = 75$

(43) (a) true (b) true (c) false (d) true (e) true
(f) true (g) false (h) false (i) false (j) false

(45) (a) $\lim_{x\to 3^-} f(x) = 3$ (b) $\lim_{x\to 3^+} f(x) = -2$

(c) $\lim_{x\to 3} f(x)$ D.N.E. (d) $f(3) = 1$

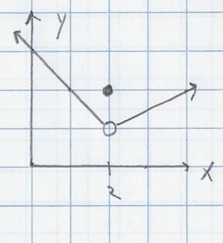
(46) (a) $\lim_{t\to -4^-} g(t) = 5$ (b) $\lim_{t\to -4^+} g(t) = 2$

(c) $\lim_{t\to -4} g(t)$ D.N.E. (d) $g(-4) = 2$

(48) (a) $\lim_{s\to -2^-} p(s) = 3$ (b) $\lim_{s\to -2^+} p(s) = 3$

(c) $\lim_{s\to -2} p(s) = 3$ (d) $p(-2) = 3$

(58)
(a)



(b) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 1$

(c) $\lim_{x\to 2} f(x) = 1$ because of part (b).

HW #1

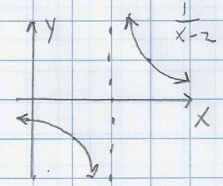
(65) $\lim_{x \to 0} x \sin x = 0 \sin 0 = 0 \cdot 0 = 0$ ◄

(67) $\lim_{x \to 0} x^2 \sin \frac{1}{x^2} = 0 \cdot \left[\begin{array}{c} \text{bounded} \\ \text{between} \\ -1 \,\&\, 1 \end{array} \right] = 0$ ◄

pg. 76

(13)

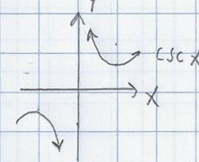
(14)



$\lim_{x \to 2^+} \frac{1}{x-2} = \infty$ ◄

$\lim_{x \to 2^-} \frac{1}{x-2} = -\infty$ ◄

(19)



$\lim_{x \to 0^+} \csc x = \infty$ ◄

pg. 76 2.2. Limits Involving Infinity

(5) (a) $\lim_{x \to \infty} \frac{3x+1}{|x|+2} = \lim_{x \to \infty} \frac{3x+1}{x+2} = \lim_{x \to \infty} \frac{3x}{x} = 3$ ◄

Dominates

(b) $\lim_{x \to -\infty} \frac{3x+1}{|x|+2} = \lim_{x \to -\infty} \frac{3x+1}{2-x} = \lim_{x \to -\infty} \frac{3x}{-x} = -3$ ◄

(c) $y = 3$ & $y = -3$ are both H.A.'s ◄

(9) $\lim_{x \to \infty} \frac{1}{x^2} \cdot (1 - \cos x) = \frac{1}{\infty} \cdot \left[\begin{array}{c} \text{bounded between} \\ 0 \,\&\, 2 \end{array} \right] = 0 \cdot [\text{bounded}] = 0$ ◄

(21) $\lim_{x \to \pm\infty} \left(2 - \frac{x}{x+1}\right) \cdot \left(\frac{x^2}{5+x^2}\right) = \lim_{x \to \pm\infty} \left(2 - \frac{x}{x}\right) \cdot \left(\frac{x^2}{x^2}\right) = (2-1)(1) = 1$ ◄

Dominates

(27) $f(x) = \frac{1}{x^2 - 4} = \frac{1}{(x+2)(x-2)} \Rightarrow x = -2$ & $x = 2$ are V.A.'s ◄

(30) $f(x) = \frac{1-x}{2x^2 - 5x - 3} = \frac{1-x}{(2x+1)(x-3)} \Rightarrow x = -0.5$ & $x = 3$ are V.A.'s ◄

$ac = 2(-3) = -6 = tu \quad b = -5 = t + u \quad t = -6 \quad u = 1$

$2x^2 - 6x + x - 3 = 2x(x-3) + 1 \cdot (x-3) = (2x+1)(x-3)$

(41) $\lim_{x \to \pm\infty} \frac{x-2}{2x^2+3x-5} = \lim_{x \to \pm\infty} \frac{x}{2x^2} = \lim_{x \to \pm\infty} \frac{1}{2x} = \frac{1}{\pm\infty} = 0$

Dominates

$\Rightarrow y = 0$ is H.A. ◄

Hw. #1

(42) $\lim\limits_{x\to\pm\infty} \dfrac{3x^2-x+5}{x^2-4}$ Dominates $= \lim\limits_{x\to\pm\infty} \dfrac{3\cancel{x^2}}{\cancel{x^2}} = 3 \Rightarrow y=3$ is H.A. ◄

Pg. 76 2.2. End-Behavior Functions of Rational Functions

(43)

2	4	0	-2	1
		8	16	28
	4	8	14	29

$f(x) = 4x^2+8x+14+\dfrac{29}{x-2} \Rightarrow E(x) = 4x^2+8x+14$ ◄

(44)

$$\begin{array}{r} -x^2-2 \\ x^2-4\overline{)-x^4+0x^3+2x^2+x-3} \\ \underline{x^4 \qquad\quad -4x^2} \\ -2x^2+x-3 \\ \underline{2x^2 \qquad -8} \\ x-11 \end{array}$$

$f(x) = -x^2-2+\dfrac{x-11}{x^2-4} \Rightarrow$

$E(x) = -x^2-2$ ◄

2.2. Relative Sizes of e^x, x^2 and $\ln x$

Supplimental

(1) (a) $\lim\limits_{x\to\infty} \dfrac{7^x}{x^2} = \dfrac{\infty}{\infty} = \dfrac{\text{big}}{\text{small}} = \infty$ ◄ Dominates (b) $\lim\limits_{x\to\infty} \dfrac{x^2}{7^x} = \dfrac{\infty}{\infty} = \dfrac{\text{small}}{\text{big}} = 0$ ◄

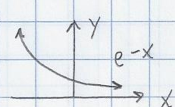
(c) $\lim\limits_{x\to\infty} \dfrac{2^x}{\log x} = \dfrac{\infty}{\infty} = \dfrac{\text{big}}{\text{small}} = \infty$ ◄ Dominates (d) $\lim\limits_{x\to\infty} \dfrac{\log x}{2^x} = \dfrac{\infty}{\infty} = \dfrac{\text{small}}{\text{big}} = 0$ ◄

(2) (a) $\lim\limits_{x\to\infty} \left(\log x - \sqrt{|x|}\right) = \infty - \infty = -\infty$ ◄

Dominates

(b) $\lim\limits_{x\to\infty} \left(1.1^x - x^{99}\right) = \infty - \infty = \infty$ ◄

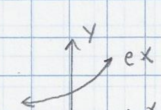
Pg. 76

(3) 

(a) $\lim\limits_{x\to\infty} \dfrac{e^{-x}}{x} = \dfrac{0}{\infty} = 0$ ◄ Dominates (b) $\lim\limits_{x\to-\infty} \dfrac{e^{-x}}{x} = \dfrac{\infty}{-\infty} = \dfrac{\text{big}}{\text{small}} = -\infty$ ◄

(c) $y=0$ is H.A. ◄

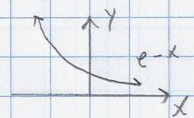
(45) $y = e^x - 2x$ (a) $x\to\infty$ e^x dominates $\Rightarrow E_R(x) = e^x$ ◄



(b) $x\to-\infty$ $-2x$ dominates $\Rightarrow E_L(x) = -2x$ ◄

(46) $y = x^2 + e^{-x}$

(a) $x \to \infty$ x^2 dominates $\Rightarrow E_R(x) = x^2$



(b) $x \to -\infty$ e^{-x} dominates $\Rightarrow E_L(x) = e^{-x}$

(69) $\lim_{x \to \infty} \frac{\ln x^2}{\ln x} = \frac{2 \cancel{\ln x}}{\cancel{\ln x}} = 2$

(70) Change of Base Formula $\Rightarrow \ln x = \frac{\log x}{\log e}$ or $\log x = \frac{\ln x}{\ln 10}$

Way #1: $\lim_{x \to \infty} \frac{\ln x}{\log x} = \frac{\frac{\log x}{\log e}}{\frac{\log x}{1}} = \log e \approx 0.434$

Way #2: $\lim_{x \to \infty} \frac{\ln x}{\log x} = \frac{\frac{\cancel{\ln x}}{1}}{\frac{\cancel{\ln x}}{\ln 10}} = \frac{1}{\ln 10} \approx 0.434$