AP CALCULUS AB

DISCONTINUITY TYPES

For problems **1** through **4**, state the discontinuity type and the x – location of the discontinuity.

1)

$$f(x) = \frac{1}{(x+4)^2}$$

Infinite discontinuity at $x = -4$

2)

$$f(x) = \cos\left(\frac{2\pi}{x-7}\right)$$

Oscillatory discontinuity at $x = 7$

3)

$$f(x) = \begin{cases} -x^2 + 4x - 1 & , \quad -\infty < x \leq 3 \\ -x^2 + 8x - 11 & , \quad 3 < x < \infty \end{cases}$$

$f(3-\varepsilon) = -(3)^2 + 4(3) - 1 = 2$

$f(3+\varepsilon) = -(3)^2 + 8(3) - 11 = 4$

ε small & positive

$\Rightarrow$ Jump discontinuity at $x = 3$

4)

$$f(x) = \frac{3x^2 + 14x + 8}{x + 4}$$

$ac = 3 \cdot 8 = 24 = 2^3 \cdot 3 = tu$

$b = 14 = t + u$

$t = 12 \quad u = 2$

$3x^2 + 12x + 2x + 8 =$

$= 3x(x+4) + 2(x+4)$

$= (3x+2)(x+4)$

$f(x) = \dfrac{(3x+2)(x+4)}{x+4} \Rightarrow$

Hole at $x = -4$

5) For

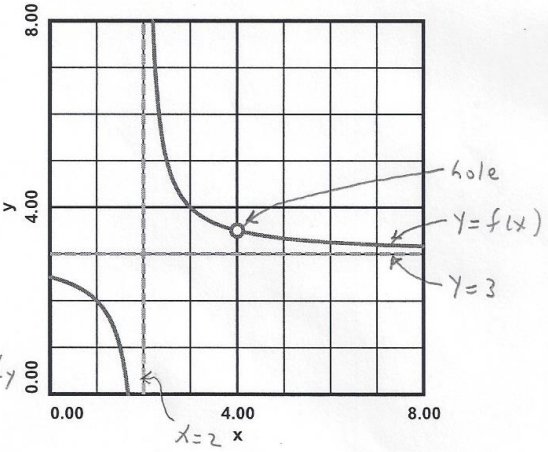
$$f(x) = \frac{3x^2 - 17x + 20}{x^2 - 6x + 8} = \frac{P(x)}{Q(x)}$$

a) find the coordinates of the hole.

b) find the horizontal and vertical asymptotes.

c) graph $y = f(x)$, the hole, and the vertical and horizontal asymptotes on the grid provided.

d) state the domain and range of $y = f(x)$.



(a)

$a \cdot c = 3 \cdot 20 = 60 = 2^2 \cdot 3 \cdot 5 = tu$

$b = -17 = t + u \quad t = -12 \quad u = -5$

$3x^2 - 12x - 5x + 20 = 3x(x-4) - 5(x-4) =$

$= (3x-5)(x-4)$

$f(x) = \dfrac{(3x-5)(x-4)}{(x-2)(x-4)}$

hole at $x = 4$

$\dfrac{3(4)-5}{4-2} = \dfrac{7}{2} = 3.5$

hole is $(4, 3.5)$

(b) H.A. is $y = 3$ V.A. is $x = 2$

(d) $x \in (-\infty, 2) \cup (2, 4) \cup (4, \infty)$

$y \in (-\infty, 3) \cup (3, 3.5) \cup (3.5, \infty)$