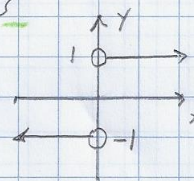
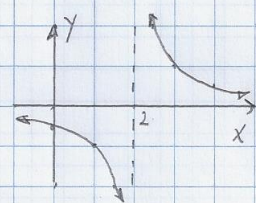
2.3. Discontinuity Types

Jumps



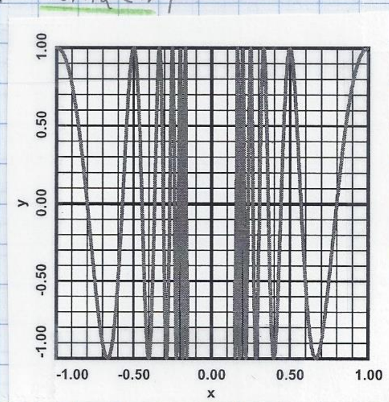
$f(x) = \dfrac{x}{|x|}$, $x \neq 0$, has a jump discontinuity at $x = 0$.

Infinite



$f(x) = \dfrac{1}{x-2}$ has an infinite discontinuity at $x = 2$

Oscillatory



Shown is the graph of $y = f(x) = \cos\left(\frac{2\pi}{x}\right)$ for $x \in [-1, -0.15] \cup [0.15, 1]$. As $x \to 0$, $y = f(x)$ oscillates more and more quickly between -1 and 1. This graph has an oscillatory discontinuity at $x = 0$.

Example #1. Find (a) $\lim\limits_{x \to 0} \cos\left(\frac{2\pi}{x}\right)$ (b) $\lim\limits_{x \to 0} x \cdot \cos\left(\frac{2\pi}{x}\right)$

SOLUTION:

(a) $\lim\limits_{x \to 0} \cos\left(\frac{2\pi}{x}\right)$ does not exist (D.N.E.) ◄ due to the oscillatory discontinuity at $x = 0$.

(b) $\lim\limits_{x \to 0} x \cdot \cos\left(\frac{2\pi}{x}\right) = 0 \cdot \left[\begin{matrix}\text{bounded between} \\ -1 \text{ and } 1\end{matrix}\right] = 0$ ◄

Holes

Example #2. For $y = f(x) = \dfrac{2x^2 - x - 21}{x^2 - x - 12} = \dfrac{P(x)}{Q(x)}$, find

(a) The coordinates of the hole.

(b) $\lim\limits_{x \to -3} f(x)$.

(c) The vertical and horizontal asymptotes of $y = f(x)$.

(d) Graph $y = f(x)$, along with its vertical & horizontal asymptotes and hole.

(e) State the domain and range of $y = f(x)$.

SOLUTION:

(a) $P(x) = 2x^2 - x - 21$ $\quad ac = 2 \cdot -21 = -42 = -2 \cdot 3 \cdot 7 = tu \quad t = 6$

$b = -1 = t + u \quad u = -7$

$P(x) = 2x^2 + 6x - 7x - 21 = 2x(x+3) - 7(x+3) = (2x-7)(x+3)$

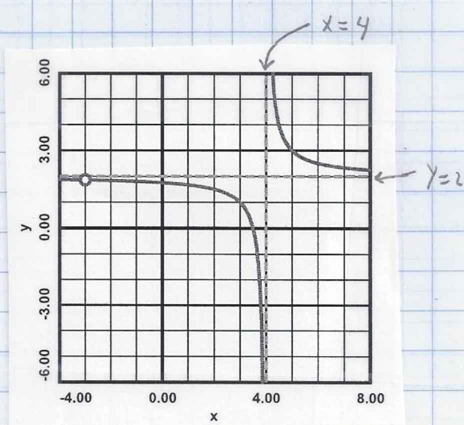
$f(x) = \dfrac{(2x-7)\cancel{(x+3)}}{(x-4)\cancel{(x+3)}}$ $\quad x = -3$ is the hole $\quad y = \dfrac{2x-7}{x-4} = \dfrac{2(-3)-7}{-3-4} = \dfrac{-13}{-7} = 1\frac{6}{7} \approx 1.86$

The hole is at $\left(-3, 1\frac{6}{7}\right)$ ◄

(b) $\lim\limits_{x \to -3} f(x) = 1\frac{6}{7}$ ◄

(c) V.A.: $x = 4$ ◄ $\quad$ H.A.: $y = 2$ ◄

(d)



(e) $x \in (-\infty, -3) \cup (-3, 4) \cup (4, \infty)$ ◄

$y \in \left(-\infty, 1\frac{6}{7}\right) \cup \left(1\frac{6}{7}, 2\right) \cup (2, \infty)$ ◄

Note: The hole at $\left(-3, 1\frac{6}{7}\right)$ is also called a removable discontinuity, because if you cancel the hole to get $f(x) \rightarrow \dfrac{2x-7}{x-4}$, then the hole has been removed.