2.3. Continuity

Definition of Continuity

If $\lim_{x \to a} f(x) = L = f(a)$, $L \neq \pm\infty$, then $y = f(x)$ is continuous at $x = a$. $y = f(x)$ is discontinuous at $x = a$ otherwise.

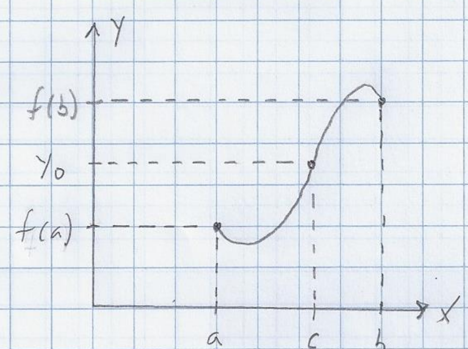
Conversely, if $y = f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

Example #1. Calculate $\lim_{x \to 3} f(x)$ for $f(x) = 5x^2 - 2x + 8$.

SOLUTION: $f(x)$ is continuous at $x = 3 \Rightarrow$

$$\lim_{x \to 3} f(x) = f(3) = 5(3)^2 - 2(3) + 8 = 47$$

Intermediate Value Theorem for Continuous Functions (IVTCF)



If $y = f(x)$ is continuous on $x \in [a, b]$, then for some y_0 between $f(a)$ and $f(b)$, there exists at least one c in (a, b) such that $f(c) = y_0$.

Example #2. For $y = f(x) = -x^2 + 8x - 11$ defined on $x \in [2, 5]$, find the value of c in $(2, 5)$ for which $y_0 = 2.75 = f(c)$.

SOLUTION:

$f(2) = 1$, $f(5) = 4$, $1 < 2.75 < 4 \Rightarrow$ IVTCF holds.

$2.75 = -c^2 + 8c - 11$, $0 = c^2 - 8c + 13.75$,

$$c = \frac{8 \pm \sqrt{8^2 - 4(1)(13.75)}}{2(1)} = \frac{8 \pm \sqrt{9}}{2} = \frac{8 \pm 3}{2}$$

$c = 5.5$ ✗ not in $(2, 5)$

$c = 2.5$ ✓

2.3. Continuity

CLASS WORK

(1) For $f(x) = 7x^2 - 2x + 15$, calculate $\lim_{x \to -2} f(x)$.

(2) For $y = f(x) = x^2 - 6x + 11$ defined on $x \in [2, 5]$, find the value of c in $(2, 5)$ such that $y_0 = f(c) = 4.25$.

SOLUTIONS

(1) $y = f(x)$ is continuous at $x = -2$ $\Rightarrow \lim_{x \to -2} f(x) = f(-2) = 7(-2)^2 - 2(-2) + 15 =$
$= 47$ ◄

(2) $f(2) = 3$, $f(5) = 6$, $3 < 4.25 < 6$ so IVTCF holds

$4.25 = c^2 - 6c + 11$, $c^2 - 6c + 6.75 = 0$

$$c = \frac{6 \pm \sqrt{6^2 - 4(1)(6.75)}}{2(1)} = \frac{6 \pm \sqrt{9}}{2} = \frac{6 \pm 3}{2}$$

$c = 4.5$ ✓ ◄

$c = 1.5$ ✗ not in $(2, 5)$