

AP CALCULUS AB

AVERAGE AND INSTANTANEOUS RATES OF CHANGE

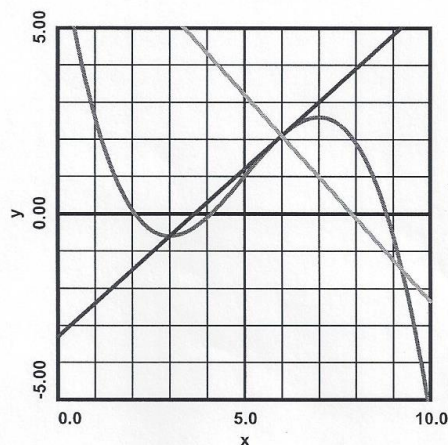
- 1) In 1997, 24.6% of U.S. adults smoked cigarettes regularly. In 2020, 13.7% of adults smoked. Find the average rate of change of percent of adults with respect to time over this time interval.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{13.7 - 24.6}{2020 - 1997} = \frac{-10.9}{23} \approx -0.474 \frac{\% \text{ adults}}{\text{yr}}$$

- 2) For $f(x) = 4x^2 - 5x + 20$, find the average rate of change of f with respect to x over $x \in [3, 7]$.

$$m_s = \frac{f(7) - f(3)}{7 - 3} = \frac{181 - 41}{7 - 3} = \frac{140}{4} = 35$$

- 3) For $f(x) = -0.1x^3 + 1.5x^2 - 6.3x + 7.5$,
- Find the slope of the function m_T at $x = 6$.
 - Find the equation of the line tangent to $y = f(x)$ at $x = 6$.
 - Find the equation of the line normal to $y = f(x)$ at $x = 6$.
 - Graph $y = f(x)$, and the tangent and normal lines, on the grid provided.



(3)

$$(a) m_T = \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h}$$

$$f(6+h) = -0.1(6+h)^3 + 1.5(6+h)^2 - 6.3(6+h) + 7.5$$

$$(6+h)^3 = 6^3 + 3 \cdot 6^2 h + 3 \cdot 6 h^2 + h^3 = 216 + 108h + 18h^2 + h^3$$

$$\begin{aligned} f(6+h) &= -0.1(216 + 108h + 18h^2 + h^3) + 1.5(36 + 12h + h^2) - 6.3(6+h) + 7.5 \\ &= -21.6 - 10.8h - 1.8h^2 - 0.1h^3 + 54 + 18h + 1.5h^2 - 37.8 - 6.3h + 7.5 \\ &= 2.1 + 0.9h - 0.3h^2 - 0.1h^3 \end{aligned}$$

$$f(6) = 2.1$$

$$f(6+h) - f(6) = 0.9h - 0.3h^2 - 0.1h^3 \quad \frac{f(6+h) - f(6)}{h} = 0.9 - 0.3h - 0.1h^2$$

$$m_T = \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h} = 0.9$$

$$(b) y = 0.9x + b, 2.1 = 0.9(6) + b, b = -3.3$$

$$y = 0.9x - 3.3$$

$$(c) m = -\frac{1}{0.9} = -\frac{10}{9}, y = -\frac{10}{9}x + b, 2.1 = -\frac{10}{9}(6) + b, b = \frac{263}{30}, y = -\frac{10}{9}x + \frac{263}{30}$$