HW #2

Supplemental:

2.3. Discontinuity Types

(1) $f(x) = \dfrac{2x^2+3x}{|x|} = (2x+3)\cdot\dfrac{x}{|x|}$ has a jump discontinuity at $x=0$ ◄

(2) $f(x) = \cos\left(\dfrac{2\pi}{x^2-4}\right)$ has oscillatory discontinuities at $x=-2$ & $x=2$ ◄

(3) $ac = 5\cdot 18 = 90 = 2\cdot 3^2\cdot 5 = tu \quad b = 33 = t+u \quad t=30 \quad u=3$

$5x^2+33x+18 = 5x^2+30x+3x+18 = 5x(x+6)+3(x+6) = (5x+3)(x+6)$

$f(x) = \dfrac{5x+3}{(5x+3)(x+6)}$ has a hole at $x = -\frac{3}{5} = -0.6$ ◄ & vertical discontinuity at $x=-6$ ◄

(4) $ac = 2(-7) = -14 = tu \quad b = 13 = t+u \quad t=14 \quad u=-1$

$2x^2+13x-7 = 2x^2+14x-x-7 = 2x(x+7)-1\cdot(x+7) = (2x-1)(x+7)$

$f(x) = \dfrac{x+3}{(2x-1)(x+7)}$ has infinite discontinuities at $x=\frac{1}{2}$ and $x=-7$ ◄

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(26) $f(x) = \dfrac{x^3-1}{(x+1)(x-1)} = \dfrac{x^2+x+1}{x+1}$ ◄

$$\begin{array}{r|rrr|r} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

(30) $f(x) = \dfrac{x^3-4x^2-11x+30}{(x+2)(x-2)} = \dfrac{x^2-2x-15}{x+2}$ ◄

$$\begin{array}{r|rrr|r} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

Supplemental:

2.3. Continuity

(5) $f(x) = x^2-8x+17$ for $x \in [3,7]$. $f(3)=2$, $f(7)=10$

$f(3) < 6 < f(7)$ so IVTCF applies. $6 = f(c) = c^2-8c+17$,

$c^2-8c+11=0 \quad c = \dfrac{8 \pm \sqrt{(-8)^2-4(1)(11)}}{2(1)} = \dfrac{8\pm\sqrt{20}}{2} = \dfrac{8\pm 2\sqrt{5}}{2} = 4\pm\sqrt{5}$

$c = 4+\sqrt{5} = 6.236$ ✓ ◄

$c = 4-\sqrt{5} = 1.764$ ✗ outside of $(3,7)$

(6) $f(x) = x^2-8x+17$ is continuous $\Rightarrow \lim\limits_{x\to 8} f(x) = f(8) = 17$ ◄

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(47) $x=3 \Rightarrow 3^2-1 = 8 = 2a(3) = 6a \quad a = \frac{8}{6} = \frac{4}{3}$

(48) $x=2 \Rightarrow 2(2)+3 = 7 = a(2)+1, \; 2a = 6, \; a = 3$

pg. 93 2.4. Average and Instantaneous Rates of Change

(1) $f(x) = x^3+1$ (a) $\frac{f(3)-f(2)}{3-2} = \frac{28-9}{1} = 19$ (b) $\frac{f(1)-f(-1)}{1--1} = \frac{2-0}{2} = 1$

(3) $f(x) = e^x$ (a) $\frac{f(0)-f(-2)}{0--2} = \frac{e^0-e^{-2}}{2} \approx 0.432$ (b) $\frac{f(3)-f(1)}{3-1} = \frac{e^3-e}{2} \approx 8.684$

(5) $f(x) = \cot x$ (a) $\frac{f(\frac{3\pi}{4})-f(\frac{\pi}{4})}{\frac{3\pi}{4}-\frac{\pi}{4}} = \frac{-1-1}{\frac{\pi}{2}} = -\frac{4}{\pi} \approx -1.273$ (b) $\frac{f(\frac{\pi}{2})-f(\frac{\pi}{6})}{\frac{\pi}{2}-\frac{\pi}{6}} = \frac{0-\sqrt{3}}{\frac{\pi}{3}} =$

$= \frac{-3\sqrt{3}}{\pi} \approx -1.654$

(10) $f(x) = x^2-4x, \; m_s(1,h) = \frac{f(1+h)-f(1)}{h},$

$f(1+h) = (1+h)^2 - 4(1+h) = 1+2h+h^2-4-4h = -3-2h+h^2, \; f(1) = -3$

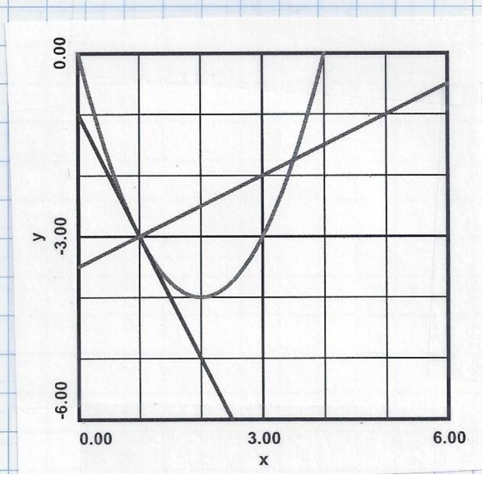
$f(1+h)-f(1) = -2h+h^2, \; \frac{f(1+h)-f(1)}{h} = -2+h,$

(a) $m_T(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}(-2+h) = -2$

(b) $y = -2x+b, \; -3 = -2(1)+b, \; b = -1, \; y = -2x-1$

(c) $y = \frac{1}{2}x+b, \; -3 = \frac{1}{2}(1)+b, \; b = -\frac{7}{2}, \; y = \frac{1}{2}x - \frac{7}{2}$

(d)



pg. 93 2.4. Velocities

(33) $f(x) = x^2 + 4x - 1$, $f(x+h) = (x+h)^2 + 4(x+h) - 1 = x^2 + 2xh + h^2 + 4x + 4h - 1 =$
$= \underbrace{x^2 + 4x - 1}_{f(x)} + 2xh + h^2 + 4h$

$f(x+h) - f(x) = 2xh + h^2 + 4h$, $\dfrac{f(x+h) - f(x)}{h} = 2x + h + 4$,

$m_T(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = 2x + 4 = 0 \Rightarrow x = -2$ $f(-2) = -5$, $(-2, -5)$

(35) $f(x) = \dfrac{1}{x-1}$, $f(x+h) - f(x) = \dfrac{1}{x+h-1} - \dfrac{1}{x-1} = \dfrac{1}{(x+h-1)} \cdot \dfrac{(x-1)}{(x-1)} - \dfrac{1}{(x-1)} \cdot \dfrac{(x+h-1)}{(x+h-1)} =$

$= \dfrac{\cancel{x} \cancel{-1} - \cancel{x} - h \cancel{+1}}{(x+h-1)(x-1)} = \dfrac{-h}{(x+h-1)(x-1)}$, $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-1}{(x+h-1)(x-1)}$,

$m_T(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \dfrac{-1}{(x-1)(x-1)} = \dfrac{-1}{(x-1)^2}$

(a) $m_T(x) = -1 = \dfrac{-1}{(x-1)^2}$, $(x-1)^2 = 1$, $x - 1 = \pm\sqrt{1} = \pm 1$, $x = 1 \pm 1$

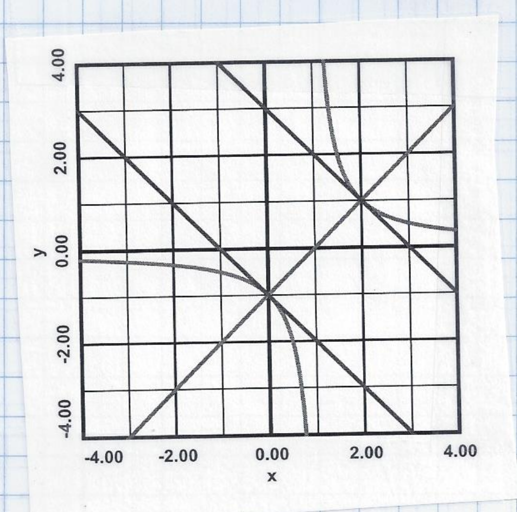
$x = 2$ $y = -x + b$, $f(2) = 1$, $1 = -2 + b$, $b = 3$, $y = -x + 3$

$x = 0$ $y = -x + b$, $f(0) = -1$, $-1 = 0 + b$, $b = -1$, $y = -x - 1$

(b) $x = 2$ $y = x + b$, $1 = 2 + b$, $b = -1$, $y = x - 1$

$x = 0$ $y = x + b$, $-1 = 0 + b$, $b = -1$, $y = x - 1$

(c)



Supplemental

7) $x = \frac{22}{9}(21t^2 - t^3)$, $x(t+h) = \frac{22}{9}\left[21(t+h)^2 - (t+h)^3\right] =$

(a) $= \frac{22}{9}\left[21(t^2+2th+h^2) - (t^3+3t^2h+3th^2+h^3)\right] =$

$= \frac{22}{9}(21t^2+42th+21h^2-t^3-3t^2h-3th^2-h^3) =$

$= \underbrace{\frac{22}{9}(21t^2-t^3)}_{x(t)} + \frac{22}{9}\left[(42t-3t^2)h + (21-3t)h^2 - h^3\right]$

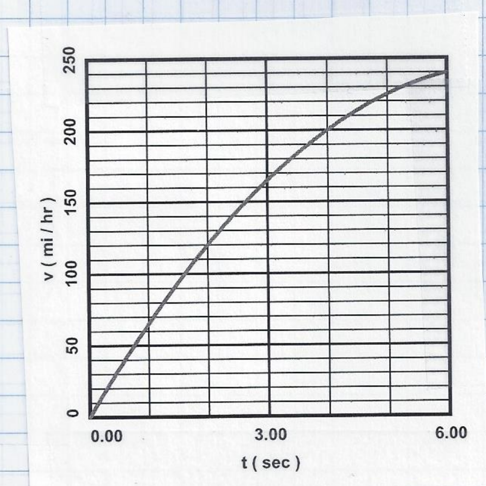
$x(t+h) - x(t) = \frac{22}{9}\left[(42t-3t^2)h + (21-3t)h^2 - h^3\right]$

$\frac{x(t+h)-x(t)}{h} = \frac{22}{9}\left[42t-3t^2+(21-3t)h-h^2\right]$

$v(t) = \lim_{h \to 0} \frac{x(t+h)-x(t)}{h} = \frac{22}{9}(42t-3t^2) = \frac{22}{3}(14t-t^2)\ \frac{ft}{sec}$ ◄

(b) $v(t) = \frac{22}{3}(14t-t^2)\frac{\cancel{ft}}{\cancel{sec}}\left(\frac{1\,mi}{5280\,\cancel{ft}}\right)\left(\frac{3600\,\cancel{sec}}{1\,hr}\right) = 5(14t-t^2)\ \frac{mi}{hr}$ ◄ ($t \equiv$ seconds)

(c)



(d) v_{max} is at $t=6$ sec ⇒

$v_{max} = v(6) = 240\ \frac{mi}{hr}$ ◄

(e) $v_{ave} = \frac{x(6)-x(0)}{6} = \frac{1320}{6} =$

$= \frac{220\,\cancel{ft}}{\cancel{sec}}\left(\frac{1\,mi}{5280\,\cancel{ft}}\right)\left(\frac{3600\,\cancel{sec}}{1\,hr}\right) =$

$= 150\ \frac{mi}{hr}$ ◄