Section 3.1 Exercises

In Exercises 1–4, use the definition

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

to find the derivative of the given function at the given value of a.

1. $f(x) = 1/x, a = 2$

2. $f(x) = x^2 + 4, a = 1$

3. $f(x) = 3 - x^2, a = -1$

4. $f(x) = x^3 + x, a = 0$

In Exercises 5–8, use the definition

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

to find the derivative of the given function at the given value of a.

5. $f(x) = 1/x, a = 2$

6. $f(x) = x^2 + 4, a = 1$

7. $f(x) = \sqrt{x+1}, a = 3$

8. $f(x) = 2x + 3, a = -1$

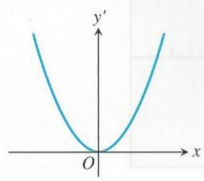
9. Find $f'(x)$ if $f(x) = 3x - 12$.

10. Find dy/dx if $y = 7x$.

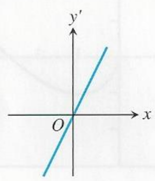
11. Find $\dfrac{d}{dx}(x^2)$.

12. Find $\dfrac{d}{dx}f(x)$ if $f(x) = 3x^2$.

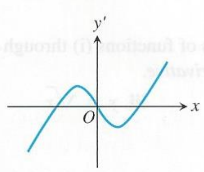
In Exercises 13–16, match the graph of the function with the graph of the derivative shown here:



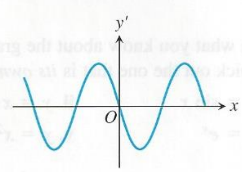
(a)



(b)

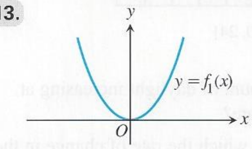


(c)

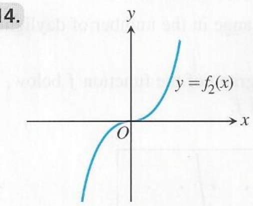


(d)

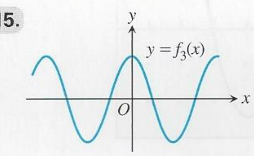
13.



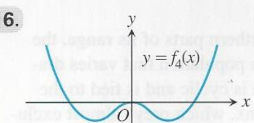
14.



15.



16.



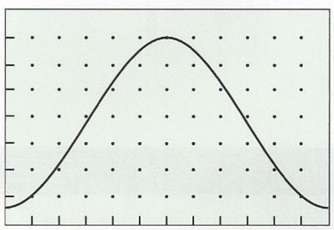
17. If $f(2) = 3$ and $f'(2) = 5$, find an equation of **(a)** the *tangent* line, and **(b)** the *normal* line to the graph of $y = f(x)$ at the point where $x = 2$.
[*Hint:* Recall that the normal line is perpendicular to the tangent line.]

18. Find the derivative of the function $y = 2x^2 - 13x + 5$ and use it to find an equation of the line tangent to the curve at $x = 3$.

19. Find the lines that are **(a)** tangent and **(b)** normal to the curve $y = x^3$ at the point $(1, 1)$.

20. Find the lines that are **(a)** tangent and **(b)** normal to the curve $y = \sqrt{x}$ at $x = 4$.

21. ***Daylight in Fairbanks*** The viewing window below shows the number of hours of daylight in Fairbanks, Alaska, on each day for a typical 365-day period from January 1 to December 31. Answer the following questions by estimating slopes on the graph in hours per day. For the purposes of estimation, assume that each month has 30 days.



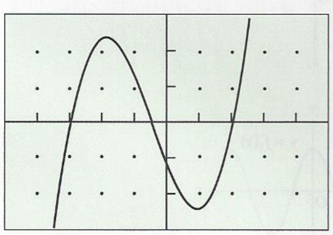
[0, 365] by [0, 24]

(a) On about what date is the amount of daylight increasing at the fastest rate? What is that rate?

(b) Do there appear to be days on which the rate of change in the amount of daylight is zero? If so, which ones?

(c) On what dates is the rate of change in the number of daylight hours positive? negative?

22. ***Graphing f′ from f*** Given the graph of the function f below, sketch a graph of the *derivative* of f.



[−5, 5] by [−3, 3]

23. ***Lynxes and Hares*** In some northern parts of its range, the snowshoe hare is known to have a population that varies dramatically over time. The variation is cyclic and is tied to the population cycle of the Canada lynx, which preys almost exclusively on snowshoe hares. A simplified version of the "predator-prey population model" is shown in Figure 3.10. Figure 3.10a shows populations of hares (blue) and lynxes (red) varying over a 9-year period in the 1800's, while Figure 3.10b shows the graph of the derivative of the hare population, determined by the slopes of the blue curve in Figure 3.10a.

(a) What is the derivative of the hare population in Figure 3.10 when the number of hares is the largest? smallest?

(b) What is the size of the hare population in Figure 3.10 when the population of hares is largest? smallest?

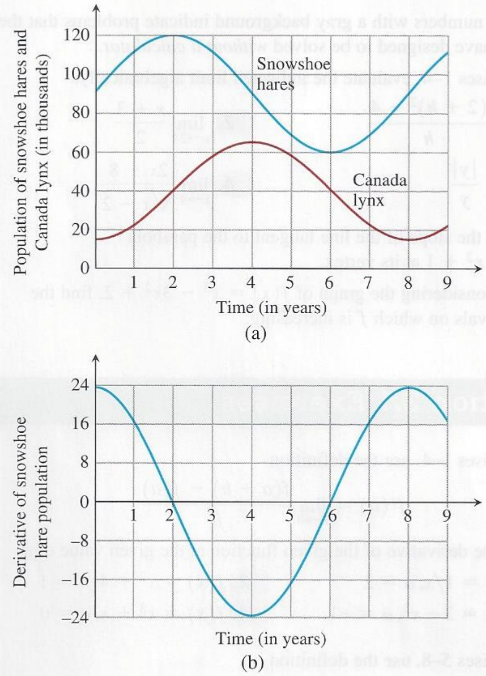


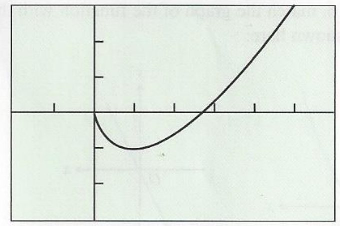
Figure 3.10 Lynxes and hares in a Canadian predator-prey food chain.

(c) Approximately how many years elapse between a peak in the prey population and a peak in the predator population?

24. Shown below is the graph of $f(x) = x \ln x - x$. From what you know about the graphs of functions (i) through (v), pick out the one that is the *derivative* of f for $x > 0$.

i. $y = e^{x-1}$ **ii.** $y = \ln x$ **iii.** $y = -\ln(x)$

iv. $y = \ln(x) - 1$ **v.** $y = 3x - 1$



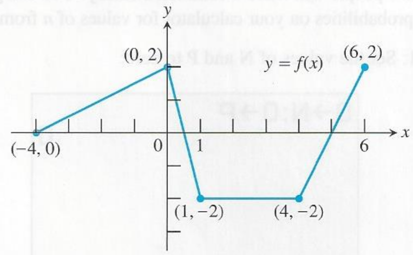
[−2, 6] by [−3, 3]

25. From what you know about the graphs of functions (i) through (v), pick out the one that is *its own derivative.*

i. $y = \sin x$ **ii.** $y = x$ **iii.** $y = \sqrt{x}$

iv. $y = e^x$ **v.** $y = x^2$

26. The graph of the function $y = f(x)$ shown here is made of line segments joined end to end.



(a) Graph the function's derivative.

(b) At what values of x between $x = -4$ and $x = 6$ is the function not differentiable?

27. ***Graphing f from f′*** Sketch the graph of a continuous function f with $f(0) = -1$ and

$$f'(x) = \begin{cases} 1, & x < -1 \\ -2, & x > -1. \end{cases}$$

28. ***Graphing f from f′*** Sketch the graph of a continuous function f with $f(0) = 1$ and

$$f'(x) = \begin{cases} 2, & x < 2 \\ -1, & x > 2. \end{cases}$$

In Exercises 29 and 30, use the data to answer the questions.

29. ***A Downhill Skier*** Table 3.3 gives the approximate distance traveled by a downhill skier after t seconds for $0 \leq t \leq 10$. Use the method of Example 5 to sketch a graph of the derivative; then answer the following questions:

(a) What does the derivative represent?

(b) In what units would the derivative be measured?

(c) Can you guess an equation of the derivative by considering its graph?

TABLE 3.3 Skiing Distances

Time t (seconds)	Distance Traveled (feet)
0	0
1	3.3
2	13.3
3	29.9
4	53.2
5	83.2
6	119.8
7	163.0
8	212.9
9	269.5
10	332.7

30. ***A Whitewater River*** Bear Creek, a Georgia river known to kayaking enthusiasts, drops more than 770 feet over one stretch of 3.24 miles. By reading a contour map, one can estimate the elevations (y) at various distances (x) downriver from the start of the kayaking route (Table 3.4).

TABLE 3.4 Elevations Along Bear Creek

Distance Downriver (miles)	River Elevation (feet)
0.00	1577
0.56	1512
0.92	1448
1.19	1384
1.30	1319
1.39	1255
1.57	1191
1.74	1126
1.98	1062
2.18	998
2.41	933
2.64	869
3.24	805

(a) Sketch a graph of elevation (y) as a function of distance downriver (x).

(b) Use the technique of Example 5 to get an approximate graph of the derivative, dy/dx.

(c) The average change in elevation over a given distance is called a *gradient.* In this problem, what units of measure would be appropriate for a gradient?

(d) In this problem, what units of measure would be appropriate for the derivative?

(e) How would you identify the most dangerous section of the river (ignoring rocks) by analyzing the graph in (a)? Explain.

(f) How would you identify the most dangerous section of the river by analyzing the graph in (b)? Explain.

31. Using one-sided derivatives, show that the function

$$f(x) = \begin{cases} x^2 + x, & x \leq 1 \\ 3x - 2, & x > 1 \end{cases}$$

does not have a derivative at $x = 1$.

32. Using one-sided derivatives, show that the function

$$f(x) = \begin{cases} x^3, & x \leq 1 \\ 3x, & x > 1 \end{cases}$$

does not have a derivative at $x = 1$.

33. Writing to Learn Graph $y = \sin x$ and $y = \cos x$ in the same viewing window. Which function could be the derivative of the other? Defend your answer in terms of the behavior of the graphs.

34. In Example 2 of this section we showed that the derivative of $y = \sqrt{x}$ is a function with domain $(0, \infty)$. However, the function $y = \sqrt{x}$ itself has domain $[0, \infty)$, so it could have a *right-hand* derivative at $x = 0$. Prove that it does not.

35. Writing to Learn Use the concept of the derivative to define what it might mean for two parabolas to be parallel. Construct equations for two such parallel parabolas and graph them. Are the parabolas "everywhere equidistant," and if so, in what sense?

Standardized Test Questions

36. True or False If $f(x) = x^2 + x$, then $f'(x)$ exists for every real number x. Justify your answer.

37. True or False If the left-hand derivative and the right-hand derivative of f exist at $x = a$, then $f'(a)$ exists. Justify your answer.

38. Multiple Choice Let $f(x) = 4 - 3x$. Which of the following is equal to $f'(-1)$?

(A) -7 **(B)** 7 **(C)** -3 **(D)** 3 **(E)** does not exist

39. Multiple Choice Let $f(x) = 1 - 3x^2$. Which of the following is equal to $f'(1)$?

(A) -6 **(B)** -5 **(C)** 5 **(D)** 6 **(E)** does not exist

In Exercises 40 and 41, let

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2x - 1, & x \geq 0. \end{cases}$$

40. Multiple Choice Which of the following is equal to the left-hand derivative of f at $x = 0$?

(A) -2 **(B)** 0 **(C)** 2 **(D)** ∞ **(E)** $-\infty$

41. Multiple Choice Which of the following is equal to the right-hand derivative of f at $x = 0$?

(A) -2 **(B)** 0 **(C)** 2 **(D)** ∞ **(E)** $-\infty$

Explorations

42. Let $f(x) = \begin{cases} x^2, & x \leq 1 \\ 2x, & x > 1. \end{cases}$

(a) Find $f'(x)$ for $x < 1$.

(b) Find $f'(x)$ for $x > 1$.

(c) Find $\lim_{x \to 1^-} f'(x)$.

(d) Find $\lim_{x \to 1^+} f'(x)$.

(e) Does $\lim_{x \to 1} f'(x)$ exist? Explain.

(f) Use the definition to find the left-hand derivative of f at $x = 1$ if it exists.

(g) Use the definition to find the right-hand derivative of f at $x = 1$ if it exists.

(h) Does $f'(1)$ exist? Explain.

43. Group Activity Using graphing calculators, have each person in your group do the following:

(a) pick two numbers a and b between 1 and 10;

(b) graph the function $y = (x - a)(x + b)$;

(c) graph the *derivative* of your function (it will be a line with slope 2);

(d) find the y-intercept of your derivative graph.

(e) Compare your answers and determine a simple way to predict the y-intercept, given the values of a and b. Test your result.

Extending the Ideas

44. Find the unique value of k that makes the function

$$f(x) = \begin{cases} x^3, & x \leq 1 \\ 3x + k, & x > 1 \end{cases}$$

differentiable at $x = 1$.

45. *Generating the Birthday Probabilities* Example 5 of this section concerns the probability that, in a group of n people, at least two people will share a common birthday. You can generate these probabilities on your calculator for values of n from 1 to 365.

Step 1: Set the values of N and P to zero:

```
0→N:0→P
                  0
```

Step 2: Type in this single, multi-step command:

```
N+1→N:1-(1-P)(366
-N)/365→P:{N,P}
```

Now each time you press the ENTER key, the command will print a new value of N (the number of people in the room) along-side P (the probability that at least two of them share a common birthday):

```
          {1 0}
  {2 .002739726}
 {3 .0082041659}
 {4 .0163559125}
 {5 .0271355737}
 {6 .0404624836}
 {7 .0562357031}
```

If you have some experience with probability, try to answer the following questions without looking at the table:

(a) If there are three people in the room, what is the probability that they all have *different* birthdays? (Assume that there are 365 possible birthdays, all of them equally likely.)

(b) If there are three people in the room, what is the probability that at least two of them share a common birthday?

(c) Explain how you can use the answer in part (b) to find the probability of a shared birthday when there are *four* people in the room. (This is how the calculator statement in Step 2 generates the probabilities.)

(d) Is it reasonable to assume that all calendar dates are equally likely birthdays? Explain your answer.

46. *Algebraic Challenge* Construct polynomial functions that fit the graphs of f and f' that are shown in Example 3 of this section.