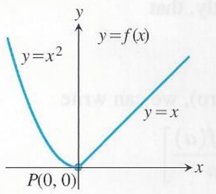
Section 3.2 Exercises

In Exercises 1–4, compare the right-hand and left-hand derivatives to show that the function is not differentiable at the point P.

1.



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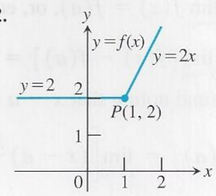

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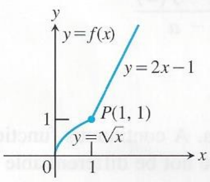

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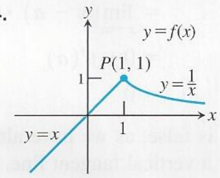


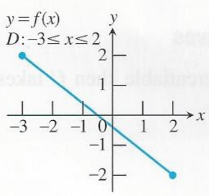
In Exercises 5–10, the graph of a function over a closed interval D is given. At what domain points does the function appear to be

(a) differentiable?

(b) continuous but not differentiable?

(c) neither continuous nor differentiable?

5.



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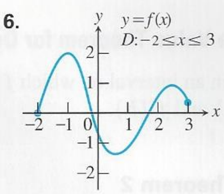

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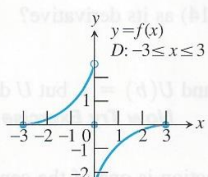

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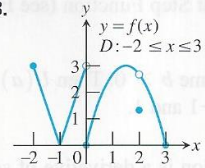

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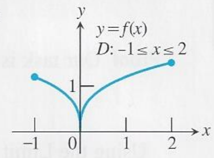

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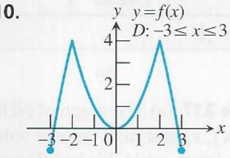

In Exercises 11–16, the function fails to be differentiable at $x = 0$. Tell whether the problem is a corner, a cusp, a vertical tangent, or a discontinuity.

11. $y = \begin{cases} \tan^{-1} x, & x \neq 0 \\ 1, & x = 0 \end{cases}$

12. $y = x^{4/5}$

13. $y = x + \sqrt{x^2} + 2$

14. $y = 3 - \sqrt[3]{x}$

15. $y = 3x - 2|x| - 1$

16. $y = \sqrt[3]{|x|}$

In Exercises 17–26, find the numerical derivative of the given function at the indicated point. Use $h = 0.001$. Is the function differentiable at the indicated point?

17. $f(x) = 4x - x^2,\ x = 0$

18. $f(x) = 4x - x^2,\ x = 3$

19. $f(x) = 4x - x^2,\ x = 1$

20. $f(x) = x^3 - 4x,\ x = 0$

21. $f(x) = x^3 - 4x,\ x = -2$

22. $f(x) = x^3 - 4x,\ x = 2$

23. $f(x) = x^{2/3},\ x = 0$

24. $f(x) = |x - 3|,\ x = 3$

25. $f(x) = x^{2/5},\ x = 0$

26. $f(x) = x^{4/5},\ x = 0$

Group Activity In Exercises 27–30, use NDER to graph the derivative of the function. If possible, identify the derivative function by looking at the graph.

27. $y = -\cos x$

28. $y = 0.25x^4$

29. $y = \dfrac{x|x|}{2}$

30. $y = -\ln|\cos x|$

In Exercises 31–36, find all values of x for which the function is differentiable.

31. $f(x) = \dfrac{x^3 - 8}{x^2 - 4x - 5}$

32. $h(x) = \sqrt[3]{3x - 6} + 5$

33. $P(x) = \sin(|x|) - 1$

34. $Q(x) = 3\cos(|x|)$

35. $g(x) = \begin{cases} (x+1)^2, & x \leq 0 \\ 2x + 1, & 0 < x < 3 \\ (4 - x)^2, & x \geq 3 \end{cases}$

36. $C(x) = x|x|$

37. Show that the function

$$f(x) = \begin{cases} 0, & -1 \le x < 0 \\ 1, & 0 \le x \le 1 \end{cases}$$

is not the derivative of any function on the interval $-1 \le x \le 1$.

38. Writing to Learn Recall that the numerical derivative (NDER) can give meaningless values at points where a function is not differentiable. In this exercise, we consider the numerical derivatives of the functions $1/x$ and $1/x^2$ at $x = 0$.

(a) Explain why neither function is differentiable at $x = 0$.

(b) Find NDER at $x = 0$ for each function.

(c) By analyzing the definition of the symmetric difference quotient, explain why NDER returns wrong responses that are so different from each other for these two functions.

39. Let f be the function defined as

$$f(x) = \begin{cases} 3 - x, & x < 1 \\ ax^2 + bx, & x \ge 1 \end{cases}$$

where a and b are constants.

(a) If the function is continuous for all x, what is the relationship between a and b?

(b) Find the unique values for a and b that will make f both continuous and differentiable.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

40. True or False If f has a derivative at $x = a$, then f is continuous at $x = a$. Justify your answer.

41. True or False If f is continuous at $x = a$, then f has a derivative at $x = a$. Justify your answer.

42. Multiple Choice Which of the following is true about the graph of $f(x) = x^{4/5}$ at $x = 0$?

(A) It has a corner.

(B) It has a cusp.

(C) It has a vertical tangent.

(D) It has a discontinuity.

(E) $f(0)$ does not exist.

43. Multiple Choice Let $f(x) = \sqrt[3]{x} - 1$. At which of the following points is $f'(a) \ne \text{NDER}(f(x), a)$?

(A) $a = 1$ **(B)** $a = -1$ **(C)** $a = 2$ **(D)** $a = -2$

(E) $a = 0$

In Exercises 44 and 45, let

$$f(x) = \begin{cases} 2x + 1, & x \le 0 \\ x^2 + 1, & x > 0. \end{cases}$$

44. Multiple Choice Which of the following is equal to the left-hand derivative of f at $x = 0$?

(A) $2x$ **(B)** 2 **(C)** 0 **(D)** $-\infty$ **(E)** ∞

45. Multiple Choice Which of the following is equal to the right-hand derivative of f at $x = 0$?

(A) $2x$ **(B)** 2 **(C)** 0 **(D)** $-\infty$ **(E)** ∞

Explorations

46. (a) Enter the expression "$x < 0$" into Y1 of your calculator using "<" from the TEST menu. Graph Y1 in DOT MODE in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.

(b) Describe the graph in part (a).

(c) Enter the expression "$x \ge 0$" into Y1 of your calculator using "≥" from the TEST menu. Graph Y1 in DOT MODE in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.

(d) Describe the graph in part (c).

47. *Graphing Piecewise Functions on a Calculator* Let

$$f(x) = \begin{cases} x^2, & x \le 0 \\ 2x, & x > 0. \end{cases}$$

(a) Enter the expression "$(X^2)(X \le 0) + (2X)(X > 0)$" into Y1 of your calculator and draw its graph in the window $[-4.7, 4.7]$ by $[-3, 5]$.

(b) Explain why the values of Y1 and $f(x)$ are the same.

(c) Enter the numerical derivative of Y1 into Y2 of your calculator and draw its graph in the same window. Turn off the graph of Y1.

(d) Use TRACE to calculate NDER(Y1, −0.1), NDER(Y1, 0), and NDER(Y1, 0.1). Compare with Section 3.1, Example 6. Did the numerical derivative get all three right?

Extending the Ideas

48. *Oscillation* There is another way that a function might fail to be differentiable, and that is by *oscillation*. Let

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & x \ne 0 \\ 0, & x = 0. \end{cases}$$

(a) Show that f is continuous at $x = 0$.

(b) Show that

$$\frac{f(0 + h) - f(0)}{h} = \sin \frac{1}{h}.$$

(c) Explain why

$$\lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$$

does not exist.

(d) Does f have either a left-hand or right-hand derivative at $x = 0$?

(e) Now consider the function

$$g(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \ne 0 \\ 0, & x = 0. \end{cases}$$

Use the definition of the derivative to show that g is differentiable at $x = 0$ and that $g'(0) = 0$.