Section 3.3 Exercises

In Exercises 1–6, find dy/dx.

1. $y = -x^2 + 3$

2. $y = \frac{x^3}{3} - x$

3. $y = 2x + 1$

4. $y = x^2 + x + 1$

5. $y = \frac{x^3}{3} + \frac{x^2}{2} + x$

6. $y = 1 - x + x^2 - x^3$

In Exercises 7–12, find the values of x for which the curve has horizontal tangents.

7. $y = x^3 - 2x^2 + x + 1$

8. $y = x^3 - 4x^2 + x + 2$

9. $y = x^4 - 4x^2 + 1$

10. $y = 4x^3 - 6x^2 - 1$

11. $y = 5x^3 - 3x^5$

12. $y = x^4 - 7x^3 + 2x^2 + 15$

13. Let $y = (x + 1)(x^2 + 1)$. Find dy/dx **(a)** by applying the Product Rule, and **(b)** by multiplying the factors first and then differentiating.

14. Let $y = (x^2 + 3)/x$. Find dy/dx **(a)** by using the Quotient Rule, and **(b)** by first dividing the terms in the numerator by the denominator and then differentiating.

In Exercises 15–22, find dy/dx. (You can support your answer graphically.)

15. $(x^3 + x + 1)(x^4 + x^2 + 1)$

16. $(x^2 + 1)(x^3 + 1)$

17. $y = \frac{2x + 5}{3x - 2}$

18. $y = \frac{x^2 + 5x - 1}{x^2}$

19. $y = \frac{(x - 1)(x^2 + x + 1)}{x^3}$

20. $y = (1 - x)(1 + x^2)^{-1}$

21. $y = \frac{x^2}{1 - x^3}$

22. $y = \frac{(x + 1)(x + 2)}{(x - 1)(x - 2)}$

23. Suppose u and v are functions of x that are differentiable at $x = 0$, and that $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$, and $v'(0) = 2$. Find the values of the following derivatives at $x = 0$.

(a) $\frac{d}{dx}(uv)$ **(b)** $\frac{d}{dx}\left(\frac{u}{v}\right)$

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ **(d)** $\frac{d}{dx}(7v - 2u)$

24. Suppose u and v are functions of x that are differentiable at $x = 2$ and that $u(2) = 3$, $u'(2) = -4$, $v(2) = 1$, and $v'(2) = 2$. Find the values of the following derivatives at $x = 2$.

(a) $\frac{d}{dx}(uv)$ **(b)** $\frac{d}{dx}\left(\frac{u}{v}\right)$

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ **(d)** $\frac{d}{dx}(3u - 2v + 2uv)$

25. Which of the following numbers is the slope of the line tangent to the curve $y = x^2 + 5x$ at $x = 3$?

i. 24 **ii.** $-5/2$ **iii.** 11 **iv.** 8

26. Which of the following numbers is the slope of the line $3x - 2y + 12 = 0$?

i. 6 **ii.** 3 **iii.** $3/2$ **iv.** $2/3$

In Exercises 27 and 28, find an equation for the line tangent to the curve at the given point.

27. $y = \frac{x^3 + 1}{2x}, x = 1$

28. $y = \frac{x^4 + 2}{x^2}, x = -1$

In Exercises 29–32, find dy/dx.

29. $y = 4x^{-2} - 8x + 1$

30. $y = \frac{x^{-4}}{4} - \frac{x^{-3}}{3} + \frac{x^{-2}}{2} - x^{-1} + 3$

31. $y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$

32. $y = 2\sqrt{x} - \frac{1}{\sqrt{x}}$

In Exercises 33–36, find the first four derivatives of the function.

33. $y = x^4 + x^3 - 2x^2 + x - 5$

34. $y = x^2 + x + 3$

35. $y = x^{-1} + x^2$

36. $y = \frac{x + 1}{x}$

37. Find an equation of the line perpendicular to the tangent to the curve $y = x^3 - 3x + 1$ at the point $(2, 3)$.

38. Find the tangents to the curve $y = x^3 + x$ at the points where the slope is 4. What is the smallest slope of the curve? At what value of x does the curve have this slope?

39. Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is parallel to the x-axis.

40. Find the x- and y-intercepts of the line that is tangent to the curve $y = x^3$ at the point $(-2, -8)$.

41. Find the tangents to *Newton's serpentine*,

$$y = \frac{4x}{x^2 + 1},$$

at the origin and the point $(1, 2)$.

42. Find the tangent to the *witch of Agnesi*,

$$y = \frac{8}{4 + x^2},$$

at the point $(2, 1)$.

43. Use the definition of derivative (given in Section 3.1, Equation 1) to show that

(a) $\frac{d}{dx}(x) = 1$.

(b) $\frac{d}{dx}(-u) = -\frac{du}{dx}$.

44. Use the Product Rule to show that

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}f(x)$$

for any constant c.

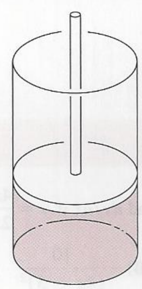
45. Devise a rule for $\frac{d}{dx}\left(\frac{1}{f(x)}\right)$.

When we work with functions of a single variable in mathematics, we often call the independent variable x and the dependent variable y. Applied fields use many different letters, however. Here are some examples.

46. ***Cylinder Pressure*** If gas in a cylinder is maintained at a constant temperature T, the pressure P is related to the volume V by a formula of the form

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2},$$

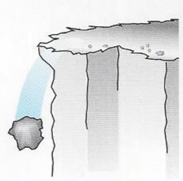
in which a, b, n, and R are constants. Find dP/dV.



47. ***Free Fall*** When a rock falls from rest near the surface of the earth, the distance it covers during the first few seconds is given by the equation

$$s = 4.9t^2$$

In this equation, s is the distance in meters and t is the elapsed time in seconds. Find ds/dt and d^2s/dt^2.



Group Activity In Exercises 48–52, *work in groups of two or three* to solve the problems.

48. ***The Body's Reaction to Medicine*** The reaction of the body to a dose of medicine can often be represented by an equation of the form

$$R = M^2\left(\frac{C}{2} - \frac{M}{3}\right),$$

where C is a positive constant and M is the amount of medicine absorbed in the blood. If the reaction is a change in blood pressure, R is measured in millimeters of mercury. If the reaction is a change in temperature, R is measured in degrees, and so on.

Find dR/dM. This derivative, as a function of M, is called the sensitivity of the body to medicine. In Chapter 5, we shall see how to find the amount of medicine to which the body is most sensitive. [*Source: Some Mathematical Models in Biology,* Revised Edition, December 1967, PB-202 364, p. 221; distributed by N.T.I.S., U.S. Department of Commerce.]

49. **Writing to Learn** Recall that the area A of a circle with radius r is πr^2 and that the circumference C is $2\pi r$. Notice that $dA/dr = C$. Explain in terms of geometry why the instantaneous rate of change of the area with respect to the radius should equal the circumference.

50. **Writing to Learn** Recall that the volume V of a sphere of radius r is $(4/3)\pi r^3$ and that the surface area A is $4\pi r^2$. Notice that $dV/dr = A$. Explain in terms of geometry why the instantaneous rate of change of the volume with respect to the radius should equal the surface area.

51. ***Orchard Farming*** An apple farmer currently has 156 trees yielding an average of 12 bushels of apples per tree. He is expanding his farm at a rate of 13 trees per year, while improved husbandry is boosting his average annual yield by 1.5 bushels per tree. What is the current (instantaneous) rate of increase of his total annual production of apples? Answer in appropriate units of measure.

52. ***Picnic Pavilion Rental*** The members of the Blue Boar society always divide the pavilion rental fee for their picnics equally among the members. Currently there are 65 members and the pavilion rents for $250. The pavilion cost is increasing at a rate of $10 per year, while the Blue Boar membership is increasing at a rate of 6 members per year. What is the current (instantaneous) rate of change in each member's share of the pavilion rental fee? Answer in appropriate units of measure.

Standardized Test Questions

53. **True or False** $\frac{d}{dx}(\pi^3) = 3\pi^2$. Justify your answer.

54. **True or False** The graph of $f(x) = 1/x$ has no horizontal tangents. Justify your answer.

55. Multiple Choice Let $y = uv$ be the product of the functions u and v. Find $y'(1)$ if $u(1) = 2$, $u'(1) = 3$, $v(1) = -1$, and $v'(1) = 1$.

(A) -4 (B) -1 (C) 1 (D) 4 (E) 7

56. Multiple Choice Let $f(x) = x - \frac{1}{x}$. Find $f''(x)$.

(A) $1 + \frac{1}{x^2}$ (B) $1 - \frac{1}{x^2}$ (C) $\frac{2}{x^3}$

(D) $-\frac{2}{x^3}$ (E) does not exist

57. Multiple Choice Which of the following is $\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$?

(A) $\frac{2}{(x-1)^2}$ (B) 0 (C) $-\frac{x^2+1}{x^2}$

(D) $2x - \frac{1}{x^2} - 1$ (E) $-\frac{2}{(x-1)^2}$

58. Multiple Choice Assume $f(x) = (x^2 - 1)(x^2 + 1)$. Which of the following gives the number of horizontal tangents of f?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Extending the Ideas

59. *Leibniz's Proof of the Product Rule* Here's how Leibniz explained the Product Rule in a letter to his colleague John Wallis:

It is useful to consider quantities infinitely small such that when their ratio is sought, they may not be considered zero, but which are rejected as often as they occur with quantities incomparably greater. Thus if we have $x + dx$, dx is rejected. Similarly we cannot have xdx and $dxdx$ standing together, as xdx is incomparably greater than $dxdx$. Hence if we are to differentiate uv, we write

$$\begin{aligned} d(uv) &= (u + du)(v + dv) - uv \\ &= uv + vdu + udv + dudv - uv \\ &= vdu + udv. \end{aligned}$$

Answer the following questions about Leibniz's proof.

(a) What does Leibniz mean by a quantity being "rejected"?

(b) What happened to $dudv$ in the last step of Leibniz's proof?

(c) Divide both sides of Leibniz's formula

$$d(uv) = vdu + udv$$

by the differential dx. What formula results?

(d) Why would the critics of Leibniz's time have objected to dividing both sides of the equation by dx?

(e) Leibniz had a similar simple (but not-so-clean) proof of the Quotient Rule. Can you reconstruct it?