Section 3.4 Exercises

1. **(a)** Write the volume V of a cube as a function of the side length s.

(b) Find the (instantaneous) rate of change of the volume V with respect to a side s.

(c) Evaluate the rate of change of V at $s = 1$ and $s = 5$.

(d) If s is measured in inches and V is measured in cubic inches, what units would be appropriate for dV/ds?

2. **(a)** Write the area A of a circle as a function of the circumference C.

(b) Find the (instantaneous) rate of change of the area A with respect to the circumference C.

(c) Evaluate the rate of change of A at $C = \pi$ and $C = 6\pi$.

(d) If C is measured in inches and A is measured in square inches, what units would be appropriate for dA/dC?

3. **(a)** Write the area A of an equilateral triangle as a function of the side length s.

(b) Find the (instantaneous) rate of change of the area A with respect to a side s.

(c) Evaluate the rate of change of A at $s = 2$ and $s = 10$.

(d) If s is measured in inches and A is measured in square inches, what units would be appropriate for dA/ds?

4. A square of side length s is inscribed in a circle of radius r.

(a) Write the area A of the square as a function of the radius r of the circle.

(b) Find the (instantaneous) rate of change of the area A with respect to the radius r of the circle.

(c) Evaluate the rate of change of A at $r = 1$ and $r = 8$.

(d) If r is measured in inches and A is measured in square inches, what units would be appropriate for dA/dr?

Group Activity In Exercises 5 and 6, the coordinates s of a moving body for various values of t are given. **(a)** Plot s versus t on coordinate paper, and sketch a smooth curve through the given points. **(b)** Assuming that this smooth curve represents the motion of the body, estimate the velocity at $t = 1.0$, $t = 2.5$, and $t = 3.5$.

5.

t (sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
s (ft)	12.5	26	36.5	44	48.5	50	48.5	44	36.5

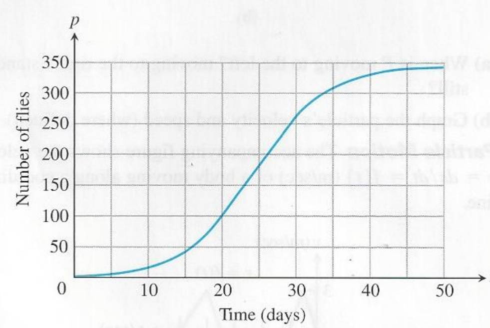
6.

t (sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
s (ft)	3.5	−4	−8.5	−10	−8.5	−4	3.5	14	27.5

7. Group Activity ***Fruit Flies*** *(Example 2, Section 2.4 continued)* Populations starting out in closed environments grow slowly at first, when there are relatively few members, then more rapidly as the number of reproducing individuals increases and resources are still abundant, then slowly again as the population reaches the carrying capacity of the environment.

(a) Use the graphical technique of Section 3.1, Example 3, to graph the derivative of the fruit fly population introduced in Section 2.4. The graph of the population is reproduced below. What units should be used on the horizontal and vertical axes for the derivative's graph?

(b) During what days does the population seem to be increasing fastest? slowest?



8. *Draining a Tank* The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$. How fast is the water running out at the end of 10 min? What is the average rate at which the water flows out during the first 10 min?

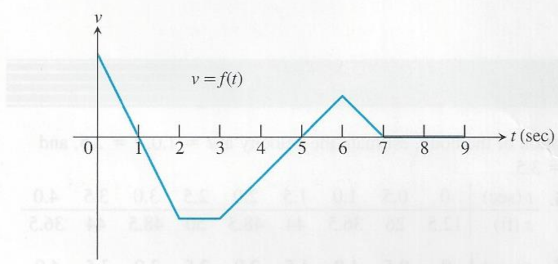
9. *Particle Motion* The accompanying figure shows the velocity $v = f(t)$ of a particle moving on a coordinate line.

(a) When does the particle move forward? move backward? speed up? slow down?

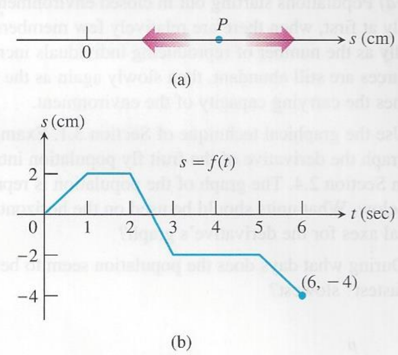
(b) When is the particle's acceleration positive? negative? zero?

(c) When does the particle move at its greatest speed?

(d) When does the particle stand still for more than an instant?



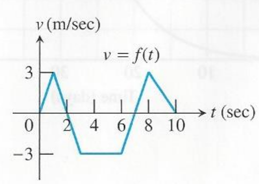
10. *Particle Motion* A particle P moves on the number line shown in part (a) of the accompanying figure. Part (b) shows the position of P as a function of time t.



(a) When is P moving to the left? moving to the right? standing still?

(b) Graph the particle's velocity and speed (where defined).

11. *Particle Motion* The accompanying figure shows the velocity $v = ds/dt = f(t)$ (m/sec) of a body moving along a coordinate line.



(a) When does the body reverse direction?

(b) When (approximately) is the body moving at a constant speed?

(c) Graph the body's speed for $0 \le t \le 10$.

(d) Graph the acceleration, where defined.

12. *Thoroughbred Racing* A racehorse is running a 10-furlong race. (A furlong is 220 yards, although we will use furlongs and seconds as our units in this exercise.) As the horse passes each furlong marker (F), a steward records the time elapsed (t) since the beginning of the race, as shown in the table below:

F	0	1	2	3	4	5	6	7	8	9	10
t	0	20	33	46	59	73	86	100	112	124	135

(a) How long does it take the horse to finish the race?

(b) What is the average speed of the horse over the first 5 furlongs?

(c) What is the approximate speed of the horse as it passes the 3-furlong marker?

(d) During which portion of the race is the horse running the fastest?

(e) During which portion of the race is the horse accelerating the fastest?

13. *Lunar Projectile Motion* A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s = 24t - 0.8t^2$ meters in t seconds.

(a) Find the rock's velocity and acceleration as functions of time. (The acceleration in this case is the acceleration of gravity on the moon.)

(b) How long did it take the rock to reach its highest point?

(c) How high did the rock go?

(d) When did the rock reach half its maximum height?

(e) How long was the rock aloft?

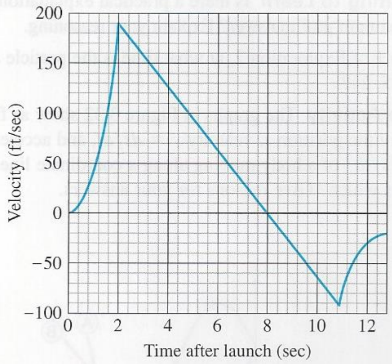
14. *Free Fall* The equations for free fall near the surfaces of Mars and Jupiter (s in meters, t in seconds) are: Mars, $s = 1.86t^2$; Jupiter, $s = 11.44t^2$. How long would it take a rock falling from rest to reach a velocity of 16.6 m/sec (about 60 km/h) on each planet?

15. *Projectile Motion* On Earth, in the absence of air, the rock in Exercise 13 would reach a height of $s = 24t - 4.9t^2$ meters in t seconds. How high would the rock go?

16. *Speeding Bullet* A bullet fired straight up from the moon's surface would reach a height of $s = 832t - 2.6t^2$ ft after t sec. On Earth, in the absence of air, its height would be $s = 832t - 16t^2$ ft after t sec. How long would it take the bullet to get back down in each case?

17. *Parametric Graphing* Devise a grapher simulation of the problem situation in Exercise 16. Use it to support the answers obtained analytically.

18. *Launching a Rocket* When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket coasts upward for a while and then begins to fall. A small explosive charge pops out a parachute shortly after the rocket starts downward. The parachute slows the rocket to keep it from breaking when it lands. This graph shows velocity data from the flight.



Use the graph to answer the following.

(a) How fast was the rocket climbing when the engine stopped?

(b) For how many seconds did the engine burn?

(c) When did the rocket reach its highest point? What was its velocity then?

(d) When did the parachute pop out? How fast was the rocket falling then?

(e) How long did the rocket fall before the parachute opened?

(f) When was the rocket's acceleration greatest? When was the acceleration constant?

19. *Particle Motion* A particle moves along a line so that its position at any time $t \geq 0$ is given by the function

$$s(t) = t^2 - 3t + 2,$$

where s is measured in meters and t is measured in seconds.

(a) Find the displacement during the first 5 seconds.

(b) Find the average velocity during the first 5 seconds.

(c) Find the instantaneous velocity when $t = 4$.

(d) Find the acceleration of the particle when $t = 4$.

(e) At what values of t does the particle change direction?

(f) Where is the particle when s is a minimum?

20. *Particle Motion* A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = -t^3 + 7t^2 - 14t + 8$ where s is measured in meters and t is measured in seconds.

(a) Find the instantaneous velocity at any time t.

(b) Find the acceleration of the particle at any time t.

(c) When is the particle at rest?

(d) Describe the motion of the particle. At what values of t does the particle change directions?

21. *Particle Motion* A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = (t - 2)^2(t - 4)$ where s is measured in meters and t is measured in seconds.

(a) Find the instantaneous velocity at any time t.

(b) Find the acceleration of the particle at any time t.

(c) When is the particle at rest?

(d) Describe the motion of the particle. At what values of t does the particle change directions?

22. *Particle Motion* A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^3 - 6t^2 + 8t + 2$ where s is measured in meters and t is measured in seconds.

(a) Find the instantaneous velocity at any time t.

(b) Find the acceleration of the particle at any time t.

(c) When is the particle at rest?

(d) Describe the motion of the particle. At what values of t does the particle change directions?

23. *Particle Motion* The position of a body at time t sec is $s = t^3 - 6t^2 + 9t$ m. Find the body's acceleration each time the velocity is zero.

24. *Finding Speed* A body's velocity at time t sec is $v = 2t^3 - 9t^2 + 12t - 5$ m/sec. Find the body's speed each time the acceleration is zero.

25. *Draining a Tank* It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank t hours after the valve is opened is given by the formula

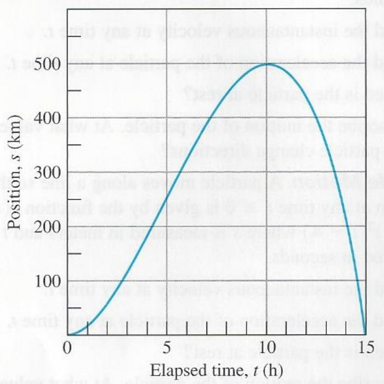
$$y = 6\left(1 - \frac{t}{12}\right)^2 \text{ m.}$$

(a) Find the rate dy/dt (m/h) at which the water level is changing at time t.

(b) When is the fluid level in the tank falling fastest? slowest? What are the values of dy/dt at these times?

(c) Graph y and dy/dt together and discuss the behavior of y in relation to the signs and values of dy/dt.

26. ***Moving Truck*** The graph here shows the position s of a truck traveling on a highway. The truck starts at $t = 0$ and returns 15 hours later at $t = 15$.



(a) Use the technique described in Section 3.1, Example 3, to graph the truck's velocity $v = ds/dt$ for $0 \leq t \leq 15$. Then repeat the process, with the velocity curve, to graph the truck's acceleration dv/dt.

(b) Suppose $s = 15t^2 - t^3$. Graph ds/dt and d^2s/dt^2, and compare your graphs with those in part (a).

27. ***Marginal Cost*** Suppose that the dollar cost of producing x washing machines is $c(x) = 2000 + 100x - 0.1x^2$.

(a) Find the average cost of producing 100 washing machines.

(b) Find the marginal cost when 100 machines are produced.

(c) Show that the marginal cost when 100 washing machines are produced is approximately the cost of producing one more washing machine after the first 100 have been made, by calculating the latter cost directly.

28. ***Marginal Revenue*** Suppose the weekly revenue in dollars from selling x custom-made office desks is

$$r(x) = 2000\left(1 - \frac{1}{x+1}\right).$$

(a) Draw the graph of r. What values of x make sense in this problem situation?

(b) Find the marginal revenue when x desks are sold.

(c) Use the function $r'(x)$ to estimate the increase in revenue that will result from increasing sales from 5 desks a week to 6 desks a week.

(d) **Writing to Learn** Find the limit of $r'(x)$ as $x \to \infty$. How would you interpret this number?

29. ***Finding Profit*** The monthly profit (in thousands of dollars) of a software company is given by

$$P(x) = \frac{10}{1 + 50 \cdot 2^{5-0.1x}},$$

where x is the number of software packages sold.

(a) Graph $P(x)$.

(b) What values of x make sense in the problem situation?

(c) Use NDER to graph $P'(x)$. For what values of x is P relatively sensitive to changes in x?

(d) What is the profit when the marginal profit is greatest?

(e) What is the marginal profit when 50 units are sold? 100 units, 125 units, 150 units, 175 units, and 300 units?

(f) What is $\lim_{x\to\infty} P(x)$? What is the maximum profit possible?

(g) **Writing to Learn** Is there a practical explanation to the maximum profit answer? Explain your reasoning.

30. In Step 1 of Exploration 2, at what time is the particle at the point $(5, 2)$?

31. **Group Activity** The graphs in Figure 3.32 show as functions of time t the position s, velocity $v = ds/dt$, and acceleration $a = d^2s/dt^2$ of a body moving along a coordinate line. Which graph is which? Give reasons for your answers.

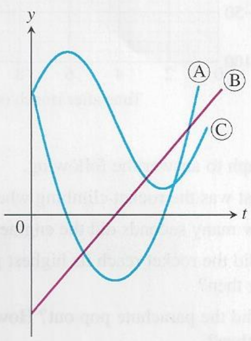


Figure 3.32 The graphs for Exercise 31.

32. **Group Activity** The graphs in Figure 3.33 show as functions of time t the position s, the velocity $v = ds/dt$, and the acceleration $a = d^2s/dt^2$ of a body moving along a coordinate line. Which graph is which? Give reasons for your answers.

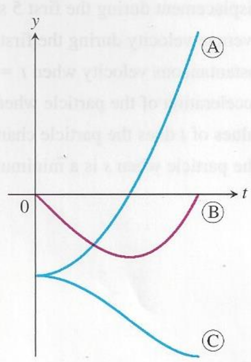


Figure 3.33 The graphs for Exercise 32.

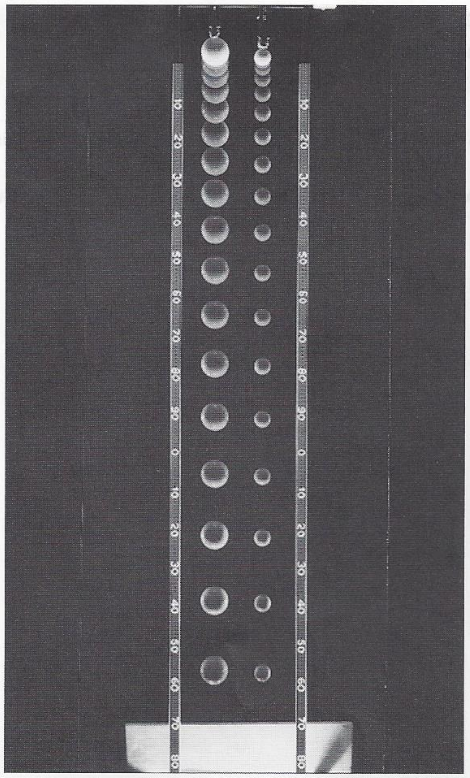


Figure 3.34 Two balls falling from rest. (Exercise 38)

33. *Pisa by Parachute* *(continuation of Exercise 18)* In 1988, Mike McCarthy parachuted 179 ft from the top of the Tower of Pisa. Make a rough sketch to show the shape of the graph of his downward velocity during the jump.

34. *Inflating a Balloon* The volume $V = (4/3)\pi r^3$ of a spherical balloon changes with the radius.

(a) At what rate does the volume change with respect to the radius when $r = 2$ ft?

(b) By approximately how much does the volume increase when the radius changes from 2 to 2.2 ft?

35. *Volcanic Lava Fountains* Although the November 1959 Kilauea Iki eruption on the island of Hawaii began with a line of fountains along the wall of the crater, activity was later confined to a single vent in the crater's floor, which at one point shot lava 1900 ft straight into the air (a world record). What was the lava's exit velocity in feet per second? in miles per hour? [*Hint:* If v_0 is the exit velocity of a particle of lava, its height t seconds later will be $s = v_0 t - 16t^2$ feet. Begin by finding the time at which $ds/dt = 0$. Neglect air resistance.]

36. Writing to Learn Suppose you are looking at a graph of velocity as a function of time. How can you estimate the acceleration at a given point in time?

37. *Particle Motion* The position (x-coordinate) of a particle moving on the line $y = 2$ is given by $x(t) = 2t^3 - 13t^2 + 22t - 5$ where t is time in seconds.

(a) Describe the motion of the particle for $t \geq 0$.

(b) When does the particle speed up? slow down?

(c) When does the particle change direction?

(d) When is the particle at rest?

(e) Describe the velocity and speed of the particle.

(f) When is the particle at the point $(5, 2)$?

38. *Falling Objects* The multiflash photograph in Figure 3.34 shows two balls falling from rest. The vertical rulers are marked in centimeters. Use the equation $s = 490t^2$ (the free-fall equation for s in centimeters and t in seconds) to answer the following questions.

(a) How long did it take the balls to fall the first 160 cm? What was their average velocity for the period?

(b) How fast were the balls falling when they reached the 160-cm mark? What was their acceleration then?

(c) About how fast was the light flashing (flashes per second)?

39. Writing to Learn Explain how the Sum and Difference Rule (Rule 4 in Section 3.3) can be used to derive a formula for *marginal profit* in terms of marginal revenue and marginal cost.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

40. True or False The speed of a particle at $t = a$ is given by the value of the velocity at $t = a$. Justify your answer.

41. True or False The acceleration of a particle is the second derivative of the position function. Justify your answer.

42. Multiple Choice Find the instantaneous rate of change of $f(x) = x^2 - 2/x + 4$ at $x = -1$.

(A) -7 **(B)** -4 **(C)** 0 **(D)** 4 **(E)** 7

43. Multiple Choice Find the instantaneous rate of change of the volume of a cube with respect to a side length x.

(A) x **(B)** $3x$ **(C)** $6x$ **(D)** $3x^2$ **(E)** x^3

In Exercises 44 and 45, a particle moves along a line so that its position at any time $t \geq 0$ is given by $s(t) = 2 + 7t - t^2$.

44. Multiple Choice At which of the following times is the particle moving to the left?

(A) $t = 0$ **(B)** $t = 1$ **(C)** $t = 2$ **(D)** $t = 7/2$ **(E)** $t = 4$

45. Multiple Choice When is the particle at rest?

(A) $t = 1$ **(B)** $t = 2$ **(C)** $t = 7/2$ **(D)** $t = 4$ **(E)** $t = 5$

Explorations

46. *Bacterium Population* When a bactericide was added to a nutrient broth in which bacteria were growing, the bacterium population continued to grow for a while but then stopped growing and began to decline. The size of the population at time t (hours) was $b(t) = 10^6 + 10^4 t - 10^3 t^2$. Find the growth rates at $t = 0$, $t = 5$, and $t = 10$ hours.

47. *Finding f from f′* Let $f'(x) = 3x^2$.

(a) Compute the derivatives of $g(x) = x^3$, $h(x) = x^3 - 2$, and $t(x) = x^3 + 3$.

(b) Graph the numerical derivatives of g, h, and t.

(c) Describe a *family* of functions, $f(x)$, that have the property that $f'(x) = 3x^2$.

(d) Is there a function f such that $f'(x) = 3x^2$ and $f(0) = 0$? If so, what is it?

(e) Is there a function f such that $f'(x) = 3x^2$ and $f(0) = 3$? If so, what is it?

48. *Pole Vaulting* In her running approach to begin her vault, a pole vaulter covered a distance of $1.5t^2$ feet in t seconds. When she planted her pole and began her ascent, she had achieved a speed of 22 feet per second. How long was her approach, both in time and in distance?

Extending the Ideas

49. *Even and Odd Functions*

(a) Show that if f is a differentiable even function, then f' is an odd function.

(b) Show that if f is a differentiable odd function, then f' is an even function.

50. *Extended Product Rule* Derive a formula for the derivative of the product fgh of three differentiable functions.