Section 3.5 Exercises

In Exercises 1–10, find dy/dx. Use your grapher to support your analysis if you are unsure of your answer.

1. $y = 1 + x - \cos x$

2. $y = 2 \sin x - \tan x$

3. $y = \dfrac{1}{x} + 5 \sin x$

4. $y = x \sec x$

5. $y = 4 - x^2 \sin x$

6. $y = 3x + x \tan x$

7. $y = \dfrac{4}{\cos x}$

8. $y = \dfrac{x}{1 + \cos x}$

9. $y = \dfrac{\cot x}{1 + \cot x}$

10. $y = \dfrac{\cos x}{1 + \sin x}$

In Exercises 11 and 12, a weight hanging from a spring (see Figure 3.38) bobs up and down with position function $s = f(t)$ (s in meters, t in seconds). What are its velocity and acceleration at time t? Describe its motion.

11. $s = 5 \sin t$

12. $s = 7 \cos t$

In Exercises 13–16, a body is moving in simple harmonic motion with position function $s = f(t)$ (s in meters, t in seconds).

(a) Find the body's velocity, speed, and acceleration at time t.

(b) Find the body's velocity, speed, and acceleration at time $t = \pi/4$.

(c) Describe the motion of the body.

13. $s = 2 + 3 \sin t$ **14.** $s = 1 - 4 \cos t$

15. $s = 2 \sin t + 3 \cos t$ **16.** $s = \cos t - 3 \sin t$

In Exercises 17–20, a body is moving in simple harmonic motion with position function $s = f(t)$ (s in meters, t in seconds). Find the jerk at time t.

17. $s = 2 \cos t$ **18.** $s = 1 + 2 \cos t$

19. $s = \sin t - \cos t$ **20.** $s = 2 + 2 \sin t$

21. Find equations for the lines that are tangent and normal to the graph of $y = \sin x + 3$ at $x = \pi$.

22. Find equations for the lines that are tangent and normal to the graph of $y = \sec x$ at $x = \pi/4$.

23. Find equations for the lines that are tangent and normal to the graph of $y = x^2 \sin x$ at $x = 3$.

24. Use the definition of the derivative to prove that $(d/dx)(\cos x) = -\sin x$. (You will need the limits found at the beginning of this section.)

25. Assuming that $(d/dx)(\sin x) = \cos x$ and $(d/dx)(\cos x) = -\sin x$, prove each of the following.

(a) $\dfrac{d}{dx} \tan x = \sec^2 x$ **(b)** $\dfrac{d}{dx} \sec x = \sec x \tan x$

26. Assuming that $(d/dx)(\sin x) = \cos x$ and $(d/dx)(\cos x) = -\sin x$, prove each of the following.

(a) $\dfrac{d}{dx} \cot x = -\csc^2 x$ **(b)** $\dfrac{d}{dx} \csc x = -\csc x \cot x$

27. Show that the graphs of $y = \sec x$ and $y = \cos x$ have horizontal tangents at $x = 0$.

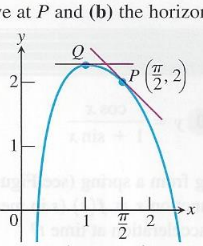
28. Show that the graphs of $y = \tan x$ and $y = \cot x$ have no horizontal tangents.

29. Find equations for the lines that are tangent and normal to the curve $y = \sqrt{2} \cos x$ at the point $(\pi/4, 1)$.

30. Find the points on the curve $y = \tan x$, $-\pi/2 < x < \pi/2$, where the tangent is parallel to the line $y = 2x$.

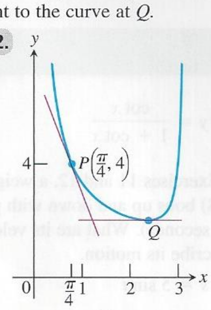
In Exercises 31 and 32, find an equation for **(a)** the tangent to the curve at P and **(b)** the horizontal tangent to the curve at Q.

31.



$y = 4 + \cot x - 2 \csc x$

32.



$y = 1 + \sqrt{2} \csc x + \cot x$

Group Activity In Exercises 33 and 34, a body is moving in simple harmonic motion with position $s = f(t)$ (s in meters, t in seconds).

(a) Find the body's velocity, speed, acceleration, and jerk at time t.

(b) Find the body's velocity, speed, acceleration, and jerk at time $t = \pi/4$ sec.

(c) Describe the motion of the body.

33. $s = 2 - 2 \sin t$ **34.** $s = \sin t + \cos t$

35. Find y'' if $y = \csc x$. **36.** Find y'' if $y = \theta \tan \theta$.

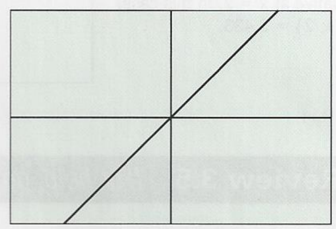
37. Writing to Learn Is there a value of b that will make

$$g(x) = \begin{cases} x + b, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$$

continuous at $x = 0$? differentiable at $x = 0$? Give reasons for your answers.

38. Find $\dfrac{d^{999}}{dx^{999}}(\cos x)$. **39.** Find $\dfrac{d^{725}}{dx^{725}}(\sin x)$.

40. ***Local Linearity*** This is the graph of the function $y = \sin x$ close to the origin. Since $\sin x$ is differentiable, this graph resembles a line. Find an equation for this line.



41. ***(Continuation of Exercise 40)*** For values of x close to 0, the linear equation found in Exercise 40 gives a good approximation of $\sin x$.

(a) Use this fact to estimate $\sin(0.12)$.

(b) Find $\sin(0.12)$ with a calculator. How close is the approximation in part (a)?

42. Use the identity $\sin 2x = 2 \sin x \cos x$ to find the derivative of $\sin 2x$. Then use the identity $\cos 2x = \cos^2 x - \sin^2 x$ to express that derivative in terms of $\cos 2x$.

43. Use the identity $\cos 2x = \cos x \cos x - \sin x \sin x$ to find the derivative of $\cos 2x$. Express the derivative in terms of $\sin 2x$.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

In Exercises 44 and 45, a weight is bobbing up and down on the end of a spring according to $s(t) = -3 \sin t$.

44. True or False The weight is traveling upward at $t = 3\pi/4$. Justify your answer.

45. True or False The velocity and speed of the weight are the same at $t = \pi/4$. Justify your answer.

46. Multiple Choice Which of the following is an equation of the tangent line to $y = \sin x + \cos x$ at $x = \pi$?

(A) $y = -x + \pi - 1$ **(B)** $y = -x + \pi + 1$
(C) $y = -x - \pi + 1$ **(D)** $y = -x - \pi - 1$
(E) $y = x - \pi + 1$

47. Multiple Choice Which of the following is an equation of the normal line to $y = \sin x + \cos x$ at $x = \pi$?

(A) $y = -x + \pi - 1$ **(B)** $y = x - \pi - 1$
(C) $y = x - \pi + 1$ **(D)** $y = x + \pi + 1$
(E) $y = x + \pi - 1$

48. Multiple Choice Find y'' if $y = x \sin x$.

(A) $-x \sin x$ **(B)** $x \cos x + \sin x$ **(C)** $-x \sin x + 2 \cos x$
(D) $x \sin x$ **(E)** $-\sin x + \cos x$

49. Multiple Choice A body is moving in simple harmonic motion with position $s = 3 + \sin t$. At which of the following times is the velocity zero?

(A) $t = 0$ **(B)** $t = \pi/4$ **(C)** $t = \pi/2$
(D) $t = \pi$ **(E)** none of these

Exploration

50. *Radians vs. Degrees* What happens to the derivatives of $\sin x$ and $\cos x$ if x is measured in degrees instead of radians? To find out, take the following steps.

(a) With your grapher in degree mode, graph

$$f(h) = \frac{\sin h}{h}$$

and estimate $\lim_{h\to 0} f(h)$. Compare your estimate with $\pi/180$. Is there any reason to believe the limit should be $\pi/180$?

(b) With your grapher in degree mode, estimate

$$\lim_{h\to 0} \frac{\cos h - 1}{h}.$$

(c) Now go back to the derivation of the formula for the derivative of $\sin x$ in the text and carry out the steps of the derivation using degree-mode limits. What formula do you obtain for the derivative?

(d) Derive the formula for the derivative of $\cos x$ using degree-mode limits.

(e) The disadvantages of the degree-mode formulas become even more apparent as you start taking derivatives of higher order. What are the second and third degree-mode derivatives of $\sin x$ and $\cos x$?

Extending the Ideas

51. Use analytic methods to show that

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = 0.$$

[*Hint:* Multiply numerator and denominator by $(\cos h + 1)$.]

52. Find A and B in $y = A \sin x + B \cos x$ so that $y'' - y = \sin x$.