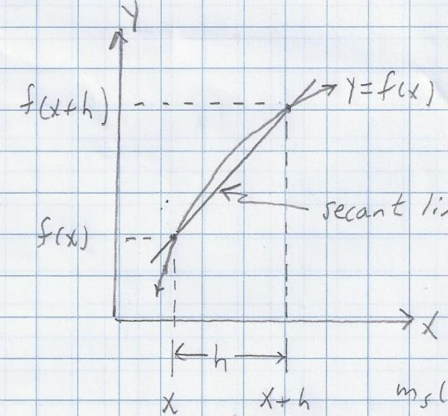
3.1. Definition of Derivative



$m_s(x,h) \equiv$ slope of the secant

$\equiv$ difference quotient

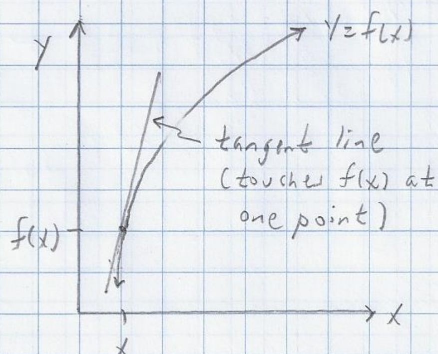
$$\boxed{m_s(x,h) = \frac{\text{rise}}{\text{run}} = \frac{f(x+h) - f(x)}{h}}$$

$m_s(x,h) \equiv$ average* rate of change of f with respect to x on $[x, x+h]$

$\equiv$ average* slope of f on $[x, x+h]$

$\boxed{h \to 0}$

* we will prove this next semester



$f'(x) \equiv$ derivative of $f(x)$

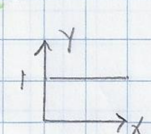
$\equiv$ instantaneous rate of change of f with respect to x at x

$\equiv$ slope of f at x.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

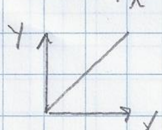
Simple Cases

(1) $f(x) = x^0 = 1$



$f'(x) = 0$

(2) $f(x) = x^1 = x$



$f'(x) = 1$

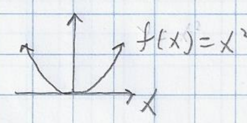
3.1. Definition of Derivative

$f(x) = x^2$

$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$, $f(x+h) - x^2 = 2xh + h^2$,

$f(x+h) - f(x) = 2xh + h^2 \qquad \dfrac{f(x+h)-f(x)}{h} = 2x + h$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0}(2x+h) = 2x$$



$f(x) = x^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Pascal's Triangle

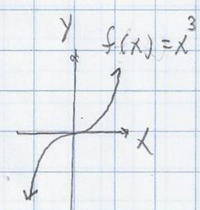
$f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$.

$f(x+h) - x^3 = 3x^2h + 3xh^2 + h^3$

$f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3$

$$\frac{f(x+h)-f(x)}{h} = 3x^2 + 3xh + h^2$$

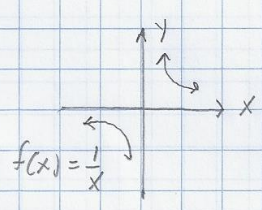
$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0}(3x^2 + 3xh + h^2) = 3x^2$$



$f(x) = x^{-1} = \frac{1}{x}$

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{1}{(x+h)} \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{(x+h)}{(x+h)} = \frac{x - x - h}{x(x+h)} = \frac{-h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-1}{x(x+h)} \qquad \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0}\left[\frac{-1}{x(x+h)}\right] = \frac{-1}{x \cdot x} = \frac{-1}{x^2}$$



3.1. Definition of Derivative

CLASS WORK

Use the definition of the derivative to calculate $f'(x)$ for

(a) $f(x) = x^4$ (b) $f(x) = \frac{1}{x^2}$

SOLUTIONS

(a) $f(x+h) = (x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$f(x+h) - x^4 = f(x+h) - f(x) = 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$$\frac{f(x+h)-f(x)}{h} = 4x^3 + 6x^2h + 4xh^2 + h^3$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \left(4x^3 + 6x^2h + 4xh^2 + h^3\right) = 4x^3 \triangleleft$$

(b) $$f(x+h) - f(x) = \frac{1}{(x+h)^2} - \frac{1}{x^2} = \frac{1}{(x+h)^2}\cdot\frac{x^2}{x^2} - \frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2} =$$

$$= \frac{x^2-(x+h)^2}{x^2(x+h)^2} = \frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2} = \frac{-2xh-h^2}{x^2(x+h)^2}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-2x-h}{x^2(x+h)^2}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2x}{x^2\cdot x^2} = -\frac{2}{x^3} \triangleleft$$